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UNIVERSITATEA **POLITEHNICA** DIN BUCUREŞTI

Facultatea de INGINERIE ELECTRICĂ

Departamentul de ELECTROTEHNICĂ

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Rezumatul Tezei de Doctorat
Summary of the PhD Thesis

Denumirea tezei în engleză

Multiphysics Modelling of Radio Frequency Micro-Electro-Mechanical-Systems

Denumirea tezei în română

Modelarea Multifizică a Întrerupătoarelor Micro-Electro-Mecanice de Radio Frecvenţe

Autor: Ing. Aurel-Sorin LUP
Conducător de doctorat: Prof. dr. ing. Daniel IOAN

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Abstract

Tema acestei teze de doctorat se referă la modelarea şi simularea atăt computaţională cât şi multifizică a dispozitivelor Micro-Electro-Mecanice (MEMS) de Radio Frecvenţă (RF). Scopul modelării este de a extrage un model compact, de ordin redus, care să poata fi simulat cu un efort de calcul cât mai mic, aceasta fiind o cerinţă foarte importantă a proiectanţilor de sisteme micro-electronice. Domeniul din care face parte cercetarea este cunoscut sub numele de Electronic Design Automation (EDA), obiectivul tezei fiind acela de a dezvolta noi metodologii şi tehnologii eficiente EDA, dedicate dispozitivelor MEMS de RF. Metodologia de cercetare aplicată în teză este bazată pe tehnici analitice, numerice şi experimentale.

Teza de doctorat este structurată în 7 capitole, primul fiind unul introductiv şi ultimul unul concluziv. Anexele şi lista bibliografică încheie teza. În Introducere se prezintă contextul tezei, importanţa şi actualitatea domeniului studiat, generalităţi despre MEMS-uri, obiectivul şi metodologia cercetării, precum şi structura lucrării. Primul capitol al lucrării descrie stadiul actual al cercetărilor din domeniul tezei de doctorat, cel al modelării dispozitivelor MEMS de radio-frecvenţă.

Capitolul al doilea este dedicat identificării principiilor teoretice ale modelării multifizice a dispozitivelor MEMS de RF. Se prezintă metodologia de modelare multifizică şi se discută conceptele teoretice care stau la baza modelării multifizice, în diferite discipline de interes pentru dispozitivele MEMS, cum sunt: Electrostatica, Electrodinamica, Mecanica structurală şi Mecanica fluidelor. În capitolul trei se prezintă studiul unui model conceptual unidimensional al unui comutator MEMS, simplificat la maxim, ca să admită o rezolvare analitică. În al patrulea capitol al lucrării se abordează problema modelării multifizice a întrerupătoarelor MEMS şi a validării modelelor extrase pe baza soluţionării lor numerice.

Capitolul cinci este dedicat studierii modelelor reduse ale comutatoarelor MEMS de RF. Acestea sunt de tipul celor prezentate în capitolul 3, dar parametrii nu sunt determinaţi pe cale analitică aproximativă, ci sunt extraşi pe baza rezultatelor numerice prezentate în capitolul 4. În capitolul şase se prezintă: concluziile generale ale lucrării, contribuţiile ei originale, subiectele deschise, ce trebuie abordate ulterior şi lista lucrărilor publicate pe perioada elaborării tezei.

Principalele contribuţii originale aduse prin teza de doctorat sunt următoarele: Analiza critică a stadiului actual al cercetărilor în domeniul modelării dispozitivelor de comutate RF MEMS. Analiza parametrică a celui mai simplu model analitic 1D al comutatoarelor RF MEMS. Dezvoltarea şi simularea cu FEM în COMSOL a modelelor numerice 2D şi 3D pentru dispozitive RF MEMS. A fost dezvoltat un algoritm eficient de extragere a unui model compact pentru comportarea statică şi dinamică a comutatoarelor RF MEMS, care a fost sintetizat în Spice. A fost dezvoltat un algoritm eficient de extragere a parametrilor modelului compact de tip TL-RLC-TL pentru comutatoarele RF MEMS. A fost dezvoltat un model parametric compact hibrid, pentru comutatoarele RF-MEMS, care conţine atât componenta de radiofrecvenţă cât şi componenta electro-mecanică multifizică.

"To my mum, with love"

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Introduction

Context: MEMS Technology and Devices

Current technological progress can be largely attributed to advances in the micro- and nano-electronics field, their applications having an important impact on the human society. It is generally accepted that the invention of the transistor was a technological step forward of great importance, but not so the actual operation of the transistor itself as the possibility of its integration at microscopic level in solid-state circuits. This inexpensive and fast process determined its tremendous success which is considered the start of the information revolution.

These new micro-electronic systems, now known as Integrated Circuits (IC) and later as Very Large Scale Integrated (VLSI) Circuits have shown a significant increase in performance, functionality and reliability, at low production costs. The success of micro-electronics have extended the researchers prospect to other areas of physics. Miniaturization principles and micro-fabrication technology with movable parts patented by the integrated circuits was subsequently applied to obtain mechanical devices with movable parts, thus micro-electro-mechanical systems (MEMS) being created.

Integrated circuits have demonstrated their applicability in all technical fields and not only. However after decades of progress, the microchip industry is in crisis, mainly because of their excessive power dissipation. Gradually, integrated circuit manufacturers have realized that a reasonable energy efficiency can not be obtained only by reducing the dimensions of a transistor. This energy performance issue is more obvious at devices and integrated circuits that contain switches and/or breakers. The problem is generated because the transistors strictly speaking, are imperfect switches (even in open state there is a leakage current).

Therefore to manufacture a competitive switch for high frequency signals, a more efficient technology, in terms of energy, had to be searched and the viable solution was the use of MEMS technology. Despite the fact that MEMS devices are not yet as fast as transistors, they compensate through energy efficiency and better performance at high frequency. One of their important advantages is they can be produced by technological processes similar the manufacture of transistors.

Micro-**E**lectro-**M**echanical **S**ystems are very small mechanical devices built on semiconductor chips having the size less than one millimeter and therefore measured in micrometers. They have appeared in research labs in the 1950ş and began to materialize as commercial products in the mid 1970ş. They have been widely used in sensors applications to measure pressure, temperature and vibration, chemical switches, gas based chromatography, light reflectors, accelerometers for airbags, vehicle controls, pacemakers and games. MEMS technology is also used to make ink jet printing heads, read/write micro-actuators heads, optical switches that reflect the light beams to corresponding output ports.

The growing demand for micro-fabricates having a superior energy performance, determined the reorientation of manufacturers to integrated circuits and hence the scientific community to the technology of micro-electro-mechanical systems, whose benefits, particularly in radio frequencies were highlighted in the last decade.

Generalities on RF-MEMS Devices

MEMS switches can be classified according to how the mechanical actuation is attained, direction of motion, electrical configuration and type of contact. The forces that mechanically actuates a RF-MEMS switches can be thermal, piezoelectric, electrostatic or magnetostatic. The most common actuation is the electrostatic one, based on electrostatic forces of attraction nature between two electrodes of different polarities, offering the advantage of low power consumption, owing to the fact that these types of switches requires electrical power only in time of commutation, not in a steady state of equilibrium. Another great advantage of these types of switches follows from compatibility with the manufacturing of integrated circuits and the ability to integrate typical MEMS devices with other micro-fabricate circuits like transistors, amplifiers or diodes.

From the construction point of view, the mechanical movement may be vertical (perpendicular to the substrate) or horizontal (parallel to the substrate). Most RF-MEMS switches made in the last 10 years have a vertical movement due to the a higher performance at high frequencies. Fundamentally, there are two constructive solutions for RF MEMS switches, bridge type and cantilever type. Electrically there are also two principles: with ohmic (resistive or galvanic) contact and with capacitive contact, being connected series or shunt in the circuit. In the case of the resistive switches, metal-to-metal contact is achieved between the two electrodes, creating a short circuit, a connection characterized by a very low contact resistance. For capacitive contact switches, the metal membrane is moved by electrostatic forces, until making contact with a isolating layer covering the other electrode, creating a high capacitance that direct the RF signal.

Objective, Research Methodology and the Layout of the Thesis

This PhD thesis has as main objective the multiphysics modelling of radio frequency micro-electro-mechanical devices, that takes in consideration all the phenomena that occur when these devices operate, such as electrostatic, structural mechanics, fluid flow and RF. The modelling is followed by the extraction of reduced order models that will allow fast and accurate simulations of electro-mechanical and RF behavior of RF-MEMS switches under several excitations.

The scientific and technical actuality of the RF-MEMS and the need for accurate and effective models required by the designers of advanced micro and nano-electronic systems, justify the importance of the research topics addressed by this PhD thesis.

The research for the PhD thesis was conducted following the **ACES** (Analytical, Computational, Experimental Solutions) methodology. According to this methodology the problems are approached in three steps. First, the solution is obtain from a simplified formulation that uses an analytical approach. Second the problem is solved by using a numerical techniques. In the end the verification of the numerical solution is done by comparison with the experimental data.

The thesis is structured in 7 chapters. The first being an introductory one, and the last a conclusive one. A short description follows.

The *Introduction* provides justification of the micro-electro-mechanical systems study, showing the importance and actuality of the research, areas of application, general presentation of RF-MEMS switches and in the end are presented the objecitves, methodology and structure of the thesis.

Chapter one, refers to the *state of the art in modelling MEMS devices.* It presents and comments the literature dedicated to modelling RF-MEMS devices

Chapter two, *Theoretical Background of the Multiphysics Modelling*, presents the general concepts, methods, techniques and technologies that are used to characterize micro-electro-mechanical systems, focusing on multiphysics modelling. There are identified the most advanced modelling techniques and the software packages used in RF-MEMS simulation, as well as the main numerical methods used in computational modelling.

Chapter three, is a detailed study case of the *one dimensional model of MEMS switches.* It explains the analytical solution of the simplest test problem that we can imagine for RF-MEMS switches. Even if the example is extremely simple based on 1D geometrical model, it is fundamental for understanding of the concepts and the physical phenomena as well as this study case practical aspects of interest. The analysis of this study case go through the five steps of modelling, starting from a general dynamic problem and ending with a particularized static one. In this analysis, the most important physical aspects and quantities are highlighted and discussed.

Chapter four, entitle *Multiphysics Modelling of MEMS Switches*, refers to multiphysics and radio frequency modelling of several test cases. First the multiphysics problem formulation is given and then the 2D and 3D numerical models are conceived. The obtained models are analyzed in static, dynamic and full wave electromagnetic regimes. The multiphysics analysis is build up from a simple static 2D model, where electrostatic field is coupled with the structural-mechanical field. Next a dynamic 3D model is analyzed, where the effect of the air flow on the movement of the membrane is considered by solving a strong coupled three field problem. After this the contact phenomena is modeled and analyzed. Studies of the influence of geometric parameters are also included in this chapter. Next an EM full wave analysis is performed, extracting the frequency response. Finally the numerical RF model is validated by comparison of simulation results with the results of experimental measurements.

Chapter five, refers to *Reduced Models for RF MEMS Switches*, it proposes effective methods for extraction the compact-lumped model of RF-MEMS device, considering multiphysics phenomena, as have been simulated numerically in the previous chapter. A mixed circuit macro-model is created, that describes the entire behavior of the switch, that include both RF and multiphysics - movement phenomena. The reduced model is then parametrized with regard to the EM behavior. The extraction methods are validated by comparison of the simulation results of the macro-model with the result of numerical simulation of the field problems.

In the last chapter, the general conclusions are drafted together with the original contributions included in this thesis and the list of the articles published by the author in the various national and international conferences and journals. Also a list of future research directions is presented here. The Appendices and References list end the thesis.

Chapter 1

State of the Art in RF-MEMS Modelling

This chapter is dedicated to the state of the art in MEMS devices modelling. The most important articles and books written to describe the research achievements in this domain, as well as an overview of the main modelling tools are depicted.

By querying the Internet databases on information about MEMS devices, Google returns over 12,800,000 results. When polling “RF MEMS switches”, Google Scholar returns more than 42,700 results and IEEE Explore returns over 1772 results, which shows that there is a very vast amount of information available. Fig.1.1 shows the evolution of the number of articles per year from 2000 to the present.

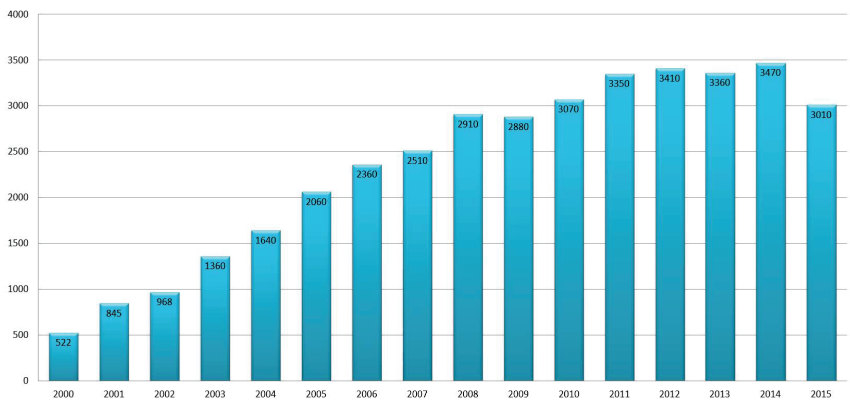

Figure 1.1: Evolution of the number of papers published per year having as a subject RF-MEMS according to `https://scholar.google.ro`.

In these circumstances, the identification and interpretation of the most relevant information is not an easy task. Thus, the data considered important for this thesis will be classified in the following categories:

- Articles and communications about RF-MEMS:
  - with general aspects (introductory, overview, applicative aspects). This category include articles and reviews describing general aspects about the characterization, structure, design, manufacturing and applications of RF-MEMS devices.

  - about modelling and simulations. This category refers to the papers describing aspect of multiphysics modelling (using original techniques and several commercial software packages) or describing the entire modelling process (ES, MEC, CFD, RF, EM) and its results, as well as papers which describe new design strategies based on EDA tools and packages.
  - extraction of Reduced Order Models. This category of articles contain methods to reduced the modelş order and methods to extract effective parameters to build an equivalent model for a MEMS switch.
  - about optimization. In this category, the mention papers present some of the new methods used in optimization of MEMS and RF-MEMS devices.
  - about experimental characterization. The papers mentioned in this category summarize some of the new methods for measurement and characterization of MEMS and RF-MEMS devices.
- PhD thesis and dissertations about RF-MEMS.
- Books about MEMS and RF-MEMS. Due to very high interest in this area, given by the large number of articles published per year, in recent years, books on general aspects of characterization of MEMS and RF-MEMS devices and their applications have been published. In this section some of the most complete books that have a high quality of information are presented.
- Software for RF-MEMS Modelling. A short description of three commercial softwares dedicated to coupled multiphysics problems is presented.

In this chapter the current state of research in the area of micro-electro-mechanical systems modelling was presented. The chapter is a critical review of the simulation and modeling techniques currently used for designing MEMS devices. Precisely multitude of techniques presented proves that there is no perfect approach. Each has its limitations, whether accuracy is unsatisfactory, either the computing effort for modeling and simulation is too high. Due to the vast information in the MEMS modeling domain the data considered important for this thesis was classified in the 4 categories: articles and communications, PhD thesis and dissertations, books and software, making a study of the publications and prior research from the recent years.

From the multitude of documentation presented, the following references are the most relevant and they influenced in a direct manner the current thesis: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11], because they present aspects that have been take on and improved in this thesis.

Chapter 2

Theoretical Background of the Multiphysics Modelling

2.1 Multiphysics Modelling

Multiphysics modelling is a procedure successfully developed in LMN - the laboratory where the thesis research was developed, which comprises the following main steps to be taken in sequence:

- *Conceptual modelling.* At this stage simplifying assumptions and neglected aspects are decided, from physical and geometric perspective.
- *Mathematical modelling.* This step includes the mathematical description of the model as a well-formulated problem.
- *The approximate analytic modelling.* It is a step that determines the simplified relationship between the input and output characteristic quantities of the modeled device, in the analytical form, by solving an approximation of the modelş equations.
- *Numerical and computational modelling.* It aims to create a dedicated algorithm able to solve the modelş equations based on the discrete reformulation of the problem, which ultimately determines how the response of the modeled object varies with respects to its excitations.
- *Verification and validation of the model.* In this stage the numerical algorithm is implemented on a computing system. A series of simulations are carried out and their results are verified by substitution in problemş equations or by comparison with results of numerical simulations based on other solving methods. The model is validated by comparison with the results of experimental measurements.
- *Reduced Order Modelling (ROM).* At this stage, using either discretized - numerical form of the equations or simulation results, simplified parametric models (pROM) are extracted, with an order much lower than that of the system of equations generated by meshing. It is desirable that these parametric models to be as compact as possible and to have an acceptable accuracy, preserving the esssential aspects of the modelş behavior.
- *Optimization.* At this stage the compact parametric models are used to identify that device from the parametric modeled class that has the best technical and/or

economic characteristics. It is a fundamental engineering activity related to optimal (re)design of components and systems.

The design of these micro-electro-mechanical devices that operate at radio frequencies requires effective procedures for modelling and software capable of simulating coupled multiphysics models. Therefor, in the thesis the following *field models* will be considered:

- *Electrostatic (ES) Model* of electric field aiming to calculate the forces acting on the mobile membrane, for its different forms. By using the elastic-mechanical model coupled to ES, the minimum ES actuation voltage/switching of the device will be determined.
- *Mechanical-Structural (MEC) Model*, from which the equilibrium position of the membrane for different actuation voltages and time evolution of its shape under the action of electrostatic forces may be determined. This model will be coupled with the electrostatic model.
- *Aerodynamic (FD) Model*, in which, besides the previous model, the air damping during the membrane movement will be considered.
- *Radio frequency (RF) Model of the EM field*, in which the frequency dependence of the S parameters is computed and a reduced order, compact circuit model will be extract for the two stable positions of the membrane (closed - DOWN and open - UP).

The advanced physical models may take into account other effects, such as the thermal aspects, the contact between the mobile electrode and the fixed one or residual mechanical stresses in the material of the membrane. The research in the field of RF-MEMS is manly directed to development at a cost as low as possible of effective models with high accuracy with a reduced complexity. Simulation and optimization of these models using computing machines, leads to the development of designs for MEMS devices with the best performance for a specific application.

Even though most aspects of the modelling activity are automated within engineering software environments as COMSOL or ANSYS, a deep understanding of the theoretical ideas and concepts used in software implementation is essential for a readable and efficient modelling activity.

2.2 Theoretical Concepts

In the general operation of MEMS devices, occur several physical phenomena that must be studied. The actuation of the device is done by applying an actuation voltage, that forms an electrostatic force, for which an electrostatic problem must be formulated and solved. For the deformation and movement of the mobile membrane a structural-mechanical problem must be formulated and solved. If the device is not packaged in vacuum, the movement suffers air damping. For this a fluid-flow problem must be formulated and solved. In this section, the fundamental equations which describe these fields, phenomena and interactions are briefly presented.

Electrostatics

In this subsection is formulated in a correct physical (conceptual) and mathematical manner the fundamental problem of the electrostatic field and are presented the methods to compute the mechanical effects of the electrostatic fields, an important component in the extraction of the multiphysics models of MEMS devices.

Electrodynamics

In this section are presented the physical (conceptual) and mathematical modelling aspects of the electromagnetic field of RF-MEMS devices.

The S parameters have an essential role in the description and characterization of the RF - microwave devices, circuits and systems. In our modeling procedure, the continuous MIMO (with infinite size of the state space) associated to EM devices is numerically modeled by FEM or FIT discretization (meshing) of Maxwell equations with EMCE boundary conditions with a finite number of states (still large). The resulted MIMO system is then reduced to compact model having a small number of states, without changing the number of terminal (number of input/output signals). The correct formulation of EM field problem with EMCE boundary condition is therefore an important step in the multiphysics modelling of RF-MEMS devices.

Structural Mechanics

Structural mechanics or Mechanics of structures is the computation of deformations, deflections, and internal forces or stresses (stress equivalents) within structures, either for design or for performance evaluation of existing structures. It is one subset of structural analysis. Structural mechanics analysis needs input data such as structural loads, the structureş geometric representation and support conditions, and the materials properties. Output quantities may include support reactions, stresses and displacements. Advanced structural mechanics include the effects of stability and non-linear behaviors. In structural mechanics every system contains three basic quantities: force, stiffness and deformation, which can be considered at different scale levels.

Fluid Mechanics

Like any real-world mathematical model, fluid mechanics takes into account several considerations regarding the studied material. These assumptions are translated into equations, which are valid only on condition that the assumptions made is real. The fluid, defined as continuously perfect environment in its structure can be deformed continuously and infinitely (so it can flow) to a shear action.

The flow is described by 7 fundamental quantities, 4 scalars, 2 vectors and 1 symmetric (3×3) tensor, in total 16 scalar components, local and instantaneous: $\mathbf{v}$ is the velocity, ρ is the mass density, p is the pressure, u is the internal energy, θ is the temperature, $\mathbf{T}$ is the stress tensor and $\mathbf{q}$ is the heat flux. Three conservation laws (Mass continuity, Conservation of momentum, Conservation of energy) are used to solve fluid dynamics problems, and may be written in integral or differential form.

2.3 Conclusions

This chapter presented the description of the multiphysics modelling procedure which comprises the following steps: Conceptual modelling, Mathematical modelling, The approximate analytic modelling, Numerical modelling, Verification and validation of the model, Extracting reduced order parametric models and in the end the Optimization.

In the general operation of MEMS devices, occur several physical phenomena that must be studied. Each physical discipline has its primitive quantities and laws. They are structural (general) and constitutive (describe material behavior). Nonlinear materials generate nonlinear equations. Local forms of laws are PDEs complied by the local and instantaneous quantities. Depending on regime, they are elliptic, parabolic or hyperbolic. Dynamical aspects and specific phenomena are described by time variations and time derivative terms in equations. Space distribution is described by spatial derivatives (rot, div, grad). Removing the metric structure of space, several systems may be described as networks, defined by their topology (So complexity is reduced!). The fundamental equations of these networks are of ODE type, w.r.t time. Between fundamental equations of several disciplines there are a series of analogies (Table 2.1), in both continuous (PDE) and discrete/circuits (ODE). So the electric circuits, described by Kirchhoffş and constitutive equations are similar to structural flow and thermal networks.

Table 2.1: Multiphysics Networks (circuits) Analogies (DISP = displacement).

General	Electrical	Mechanical	Fluidic	Thermal
Effort, e	Voltage, V	Force, F	Pressure, P	Temp. diff., ΔT
Flow, f	Current, I	Velocity, v	Vol. flow rate, Q	Heat flow
DISP, q	Charge, Q	DISP, x	Volume, V	Heat, Q
Momentum p	-	Momentum, p	Pressure Momentum, Γ	-
Resistance	Resistor, R	Damper, b	Fluidic resistance, R	Thermal resistance, R
Capacitance	Capacitor, C	Spring, k	Fluid capacitance, C	Heat capacity, mcp
Inertance	Inductor, L	Mass, m	Inertance, M	-
Node law	KCL	Continuity of space	Mass conservation	Heat energy conservation
Mesh law	KVL	Newtonş 2^{nd} law	Pressure is relative	Temperature is relative

The objective of the RF-MEMS switch is to control the flow of the RF signal, therefore its electromagnetic analysis is essential. As we expected it is dependent on the mechanical/geometrical state of the switch. The actuation of the device is done by applying an actuation voltage, that generates an electrostatic force, this is why an electrostatic problem must be formulated and solved. For the deformation and movement of the flexible switchş membrane a structural-mechanical problem must be formulated and solved. If the device is not packaged in vacuum, the movement suffers air damping. For this a fluid-flow problem must be formulated and solved. Because all the phenomena influence each other, their interactions must be also formulated as a coupled multiphysics problem. Thereby, the correct mathematically formulation of the electrostatic, electrodynamic, structural mechanics and fluid flow field problems and their couplings must to be described, based of deep understanding of the device operation. The theoretical background of this procedure was presented in this chapter. The knowledge presented here allows a proper formulation of multiphysics models and their description to the computational environments and dedicated CAD/CAE software packages, as well as their simulation by several numerical methods and techniques.

Chapter 3

One Dimensional Models of MEMS Switches

The aim of this step of research is the dynamic and static analysis of the simplest models that we can imagine for RF-MEMS Switches, the 1D one of a *parallel plane capacitor with a mobile armature suspended by a spring.*

Even if this example is extremely simple, it will allow the understanding of the concepts and the physical phenomena as well as the practical aspects that are of maxim interest in engineering.

Regardless the exact geometry of the MEMS switch, there is a mobile, elastic plate, which is deformed by electrostatic forces. If the exact shape of the plate is neglected we may consider it as plane capacitor. This parallel plate capacitor has the armatures (made from a rigid conducting material) placed horizontally, the bottom armature being fixed, and the top armature being suspended by means of a virtual elastic spring (Fig.3.1). The device is placed in vacuum or in a gas. Supplementary, on the bottom armature there may be a thin dielectric layer (Fig.3.2). An electric actuation voltage is applied between the two plates. These figures describe the geometric object, this is why no forces are drawn.

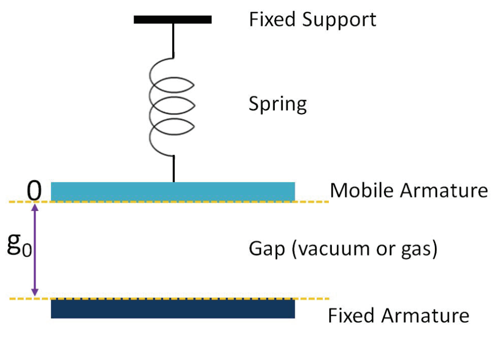

Figure 3.1: Resistive Switch.

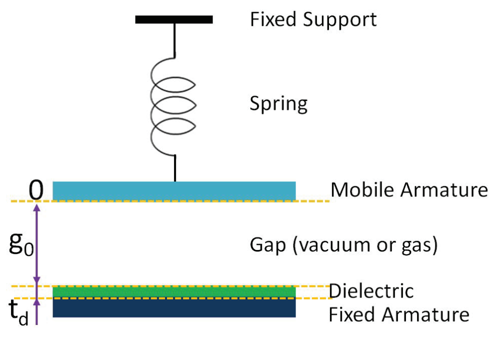

Figure 3.2: Capacitive Switch.

When a voltage is applied between the capacitor plates, an electric force appears which moves the mobile plate. If the force is big enough the movement will take until a mechanical contact between the two plates take place (DOWN stable state). When the voltage is zeroed, the system moves back to the initial position (UP stable state) due to the elastic forces of the spring. During the movement which is considered rectilinear, there is a drag force due to the relative movement of the mobile plate with respect to the gas that surrounds it. It is obvious that the movement is non-uniform (the velocity is not constant, the acceleration is non-zero, so when writing the equilibrium equations in a reference system attached to the mobile plate, an inertial force has to be considered).

3.1 Dynamic Regime

The mobile armature moves due to electrostatic forces, but movement is influenced also by other aspects such as inertial force, elastic force, damping due to internal friction or air, as well as interaction with the contact. Weight may be neglected because its gravitational effect is nonessential. Because the model of the switch is dynamic, mechanical movement is the main phenomenon that occurs, its equations underlying the model. So we start from the fundamental equation of mechanics, Newtonş second law:

$$F = ma \Rightarrow F = m\frac{\mathrm{d}v}{\mathrm{d}t}; \quad v = \frac{\mathrm{d}z}{\mathrm{d}t} \tag{3.1}$$

where: m is the mass of the body in motion, F is the total force acting on the body and a is the acceleration of the body.

State equations of motion are therefore:

$$\frac{\mathrm{d}z}{\mathrm{d}t} = v; \quad \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{F}{m}; \tag{3.2}$$

The excitation of the system is the force *F(t)* which is time-dependent. If it also depends to the state variables *F(z, v, t)* in a nonlinear way, then we have a nonlinear system. The main components of the force are $F = F_{ES} + F_c + F_e + F_a$, where:

- $F_{\mathrm{ES}}(u,z) = \frac{1}{2}\frac{\mathrm{d}W}{\mathrm{d}z} = \frac{u^2}{2}\frac{\mathrm{d}C}{\mathrm{d}z} = \frac{1}{2}\frac{\varepsilon_0 A u^2}{(g_0 - z + \frac{t_d}{\varepsilon_0})^2}$ is the electrostatic force.
- $F_c(z,v)$ is the contact force, which describes the interaction with the contact. A viscoelastic model can be used for it, similar to those of the mobile armature, but with different values of the constants, ie $F_c = k'(z - g_0) - b'v$, model that only applies if the armature is in contact, $z > g_0$, otherwise $F_c = 0$.
- $F_e = -kz - k_s z^3$ is the elastic force, depending in the simplest model to the displacement z, where k is the linear elasticity coefficient and k_s is a non-linear (cubic) elasticity coefficient.
- $F_a = -bv$ is the damping force, proportional in the simplest model to the velocity v, where b is the damping coefficient.

Consequently, the following motion state equations are obtained:

$$\begin{cases} \frac{\mathrm{d}z}{\mathrm{d}t} = v; \\ m\frac{\mathrm{d}v}{\mathrm{d}t} = F_{\mathrm{ES}}(u,z) + F_c(z,v) - kz - kz^3 - bv. \end{cases} \tag{3.3}$$

Analytical solutions may be obtained in the following simplifying hypothesis: $k_s = 0, F_c = 0$ (study of the movement until it reaches the contact), b is constant and F_{ES} is constant (e.g. equal to the value for $z = 0$). Under the assumption $F_{\mathrm{ES}} = E_{\mathrm{ES-ct}}$, we can solve linear second order ordinary differential the equation and get an idea of the order of magnitude of the involved quantities:

$$\begin{cases} m\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} + b\frac{\mathrm{d}z}{\mathrm{d}t} + kz = F_{\mathrm{ES-ct}} \\ z(0) = 0, \frac{\mathrm{d}z}{\mathrm{d}t|_0} = 0 \end{cases} \tag{3.4}$$

The study case presented above will be analyzed by using *MATLAB*, *APLAC* and *SPICE*. The numerical values we consider are these given in the APLAC documentation. The problem formulation as in the APLAC documentation refers to a mechanical resonator whose physical appearance is not important, having the mass M of 10 mg, spring coefficient of $k = 3553$ N/m and quality factor Q of 3. A stub limits the maximum displacement of the mass $z_{\max}(d_{\max}) = 5\mu$m, the air gap $g_0(d) = 10\mu$m and the surface area $A(l \times w)$ is 100 mm^2.

In *MATLAB*, the ODE numerical procedures accept only first order equations or systems of equations. This is why our second order ODE equation has to be reformulated in the state form, as a first order system:

$$m\frac{\mathrm{d}^2 z}{\mathrm{d}t^2} + b\frac{\mathrm{d}z}{\mathrm{d}t} + kz + k_s z^3 = F_{\mathrm{ES}} + F_c \Rightarrow \begin{cases} \frac{\mathrm{d}z}{\mathrm{d}t} = v; \\ \frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{b}{m}v - \frac{k}{m}z - \frac{k_s}{m}z^3 - \frac{F_{\mathrm{ES}}}{m} - \frac{F_c}{m}. \end{cases} \tag{3.5}$$

The results obtained are shown in Fig.3.3, on the right side is presented the velocity dependence on time and on the left side is presented the displacement dependence on time.

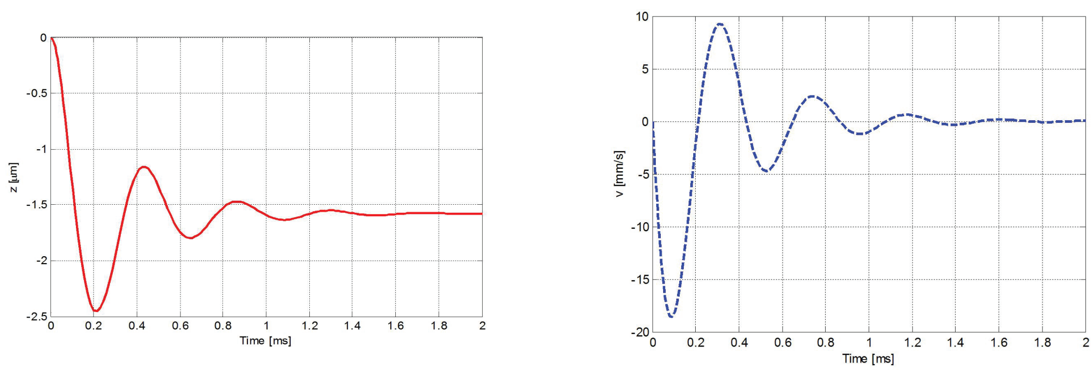

Figure 3.3: Time dependence of the displacement and velocity resulted from MATLAB analysis.

The results from the simulation in APLAC are presented in Fig.3.4.

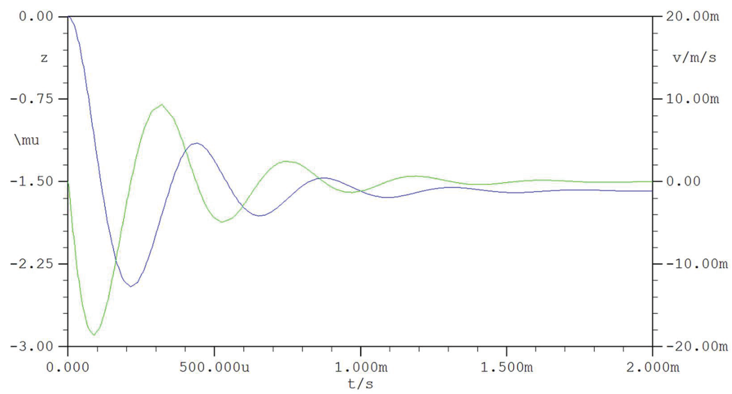

Figure 3.4: Time dependence of the displacement and velocity resulted from APLAC analysis.

In Fig.3.5 is presented the SPICE circuit model and the time dependence *V(n003)* which represents the displacement z in μm.

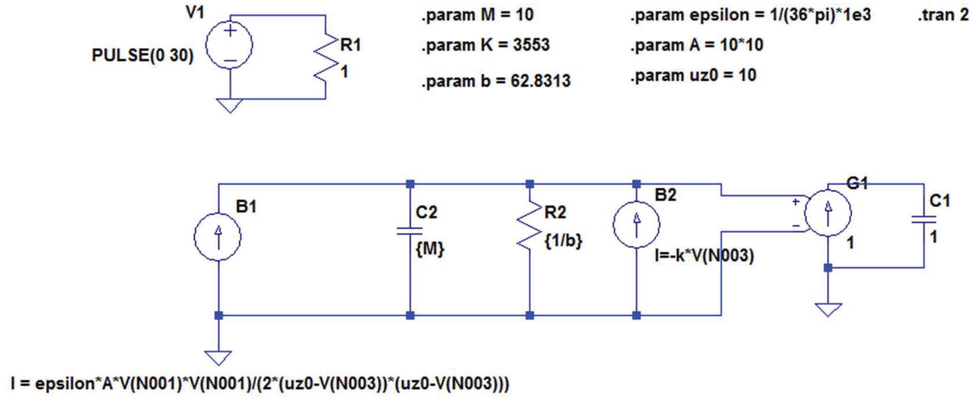

Figure 3.5: SPICE circuit model.

In order to be compatible with the APLAC notation, -z will be displayed instead of z. So -*V(n003)* is the displacement in the APLAC notation and -*V(n002)* is the velocity (Fig.3.6).

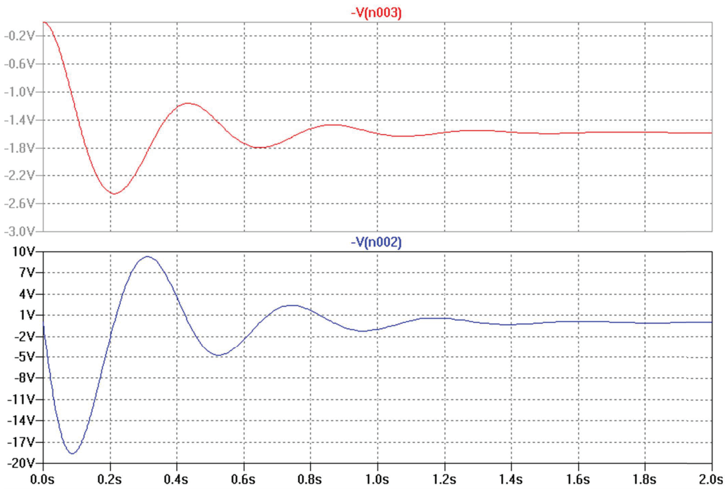

Figure 3.6: Time dependence of the displacement (UP) and velocity (DOWN) resulted from SPICE analysis.

Since we validated the SPICE models and MATLAB procedures, we can use them to better understand the influence of various parameters. A parametric analysis was made using the simple APLAC example and change one parameter at a time, under a voltage step excitation of 30 V (which is less than pull-in voltage, so the system moves from the initial state, but the contact will not be reached). The switching time and the release time was also analyzed.

3.2 Static Regime

In this regime the time dependencies of the quantities are neglected (d/dt = 0) and is assumed that energies transfers are non existent, neither electrostatic nor elastostatic, i.e. top armature is in a equilibrium position, is immobile.

In static regime, assuming that the elastic constant is linear, the movement equation takes the form:

$$-kz\mathbf{k} + \mathbf{F}_\varepsilon = 0. \tag{3.6}$$

The equations being algebraic, there is no need to impose any boundary conditions and initial conditions. Obviously by substituting the expression of the electrostatic forces, it results a third order algebraic equation, having the form:

$$k(g_0 - g) = \frac{\varepsilon_0 A V^2}{2\left(g + \frac{t_d}{\varepsilon_r}\right)^2}. \tag{3.7}$$

3.2.1 Analytical Modelling in Static Regime

The considered study case [1] is a capacitor with area of the armature $A = LW$, where $L = 100\mu$m is the length and $W = 100\mu$m is the width of the armature. The initial distance is $g_0 = 3\mu$m and the elastic constant of the spring is $k = 10$N/m. The actuation voltage used is in the range of 0 to 40 volts. The simplicity of the problem allows an exact analytical modelling of the problem.

Resistive Case. For this case there is no isolation layer between the armatures $t_d = 0$, the nonlinear equation has the form:

$$f_{rez}(g) = g^3 - g_0 g^2 + \frac{\varepsilon_0 A V^2}{2k}. \tag{3.8}$$

The nonlinear equation will always have a negative solution, which has no physical significance and other two solutions, from which one or non real solutions, everything depends on the actuation voltage. The graphical representation of this function for the values specified above for different applied voltages is presented in Fig.3.7 - Left. A pull-in voltage $V_{\text{pi}} = 30.08$ V and the displacement of $g_{\text{pi}} = 2\mu$n.

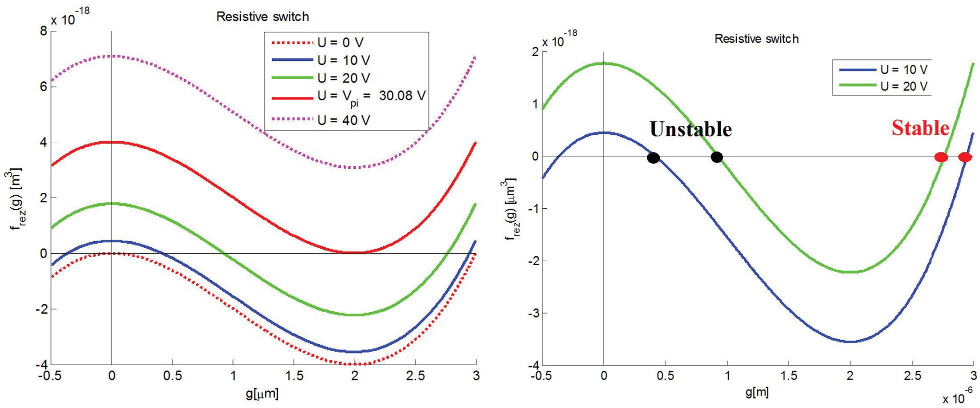

Figure 3.7: Left: Dependency of the nonlinear function on the distance between armatures for different values of the applied voltage. Right: Stable and unstable positions.

Capacitive Case. For this case the isolation layer between the armatures has a thickness $t_d \neq 0$, the nonlinear equation has the form:

$$f_{cap}(g) = (g - g_0)\left(g + \frac{t_d}{\varepsilon_r}\right)^2 + \frac{\varepsilon_0 A V^2}{2k}. \tag{3.9}$$

The local minimum point that gives the value of the pull-in voltage is smaller then two thirds of the initial gap. The expressions for the pull-in voltage and the displacement at this value are:

$$V_{pi} = \sqrt{\frac{8k}{27\varepsilon_0 A}\left(g_0 + \frac{t_d}{\varepsilon_r}\right)^3} \text{ and } g_{pi} = \frac{2g_0}{3} - \frac{t_d}{3\varepsilon_r}. \quad (3.10)$$

To make a comparison with the resistive case, in the 3μm initial gap of the resistive switch an isolation layer is inserted whose relative permittivity has the value of 7.5 typical for Si_3N_4.

The analysis of the nonlinear dependence of actuation voltage shows that the insertion of the dielectric between the armature has the effect of lowering the pull-in voltage (Fig.3.8 - Left). For this configuration with $g_0 = 2.5\mu$m and $t_d = 0.5\mu$m results a $V_{\mathrm{pi}} = 23.8$ V with a displacement $g_{\mathrm{pi}} = 1.64\mu$m. For this example the thickness of the isolator is a bit exaggerated, usually $t_d = 0.1\mu$m, resulting a initial gap $g_0 = 2.9\mu$m and $V_{\mathrm{pi}} = 28.79$ V with a displacement $g_{\mathrm{pi}} = 1.93\mu$m (Fig.3.8 - Right).

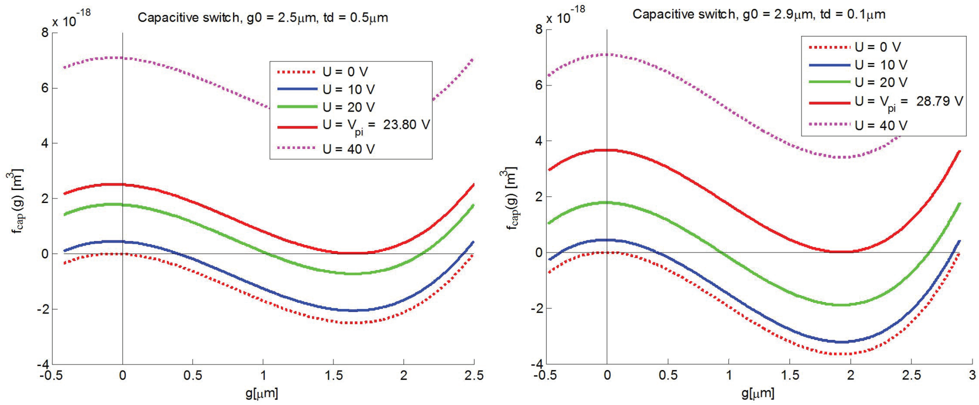

Figure 3.8: Dependence of nonlinear function f_{cap} of the distance between armatures for different values of the applied voltage. Left: Configuration with $g_0 = 2.5\mu$m and $t_d = 0.5\mu$m. Right: Configuration with $g_0 = 2.9\mu$m and $t_d = 0.1\mu$m.

The dependence of the stable and unstable solutions with the applied voltage (Fig.3.9 - Left) was similar to the resistive case applying the expression:

$$V = \sqrt{\frac{2k}{\varepsilon_0 A}(g_0 - g)}\left(g + \frac{t_d}{\varepsilon_r}\right)^2. \quad (3.11)$$

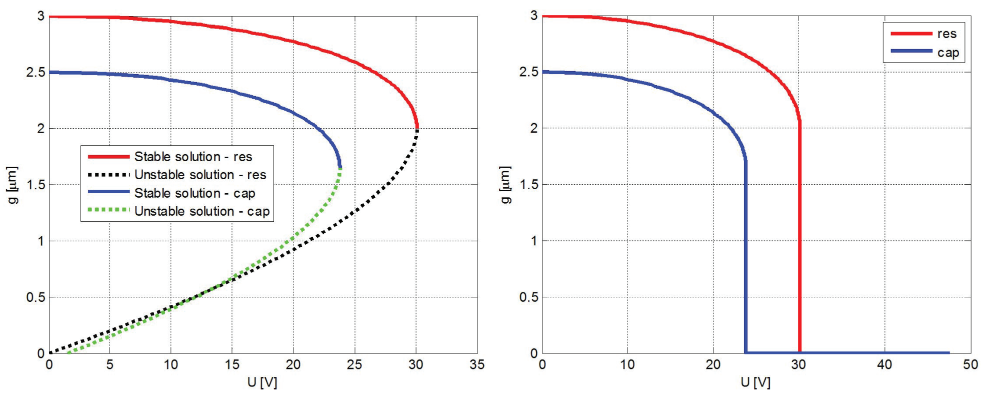

Figure 3.9: Left: Dependency of the gap between the armatures of the applied voltage. Right: Real dependency.

A more correct representation is given in Fig.3.9 - Right, for an applied voltage greater then V_{pi}; the switch is actuated and the gap between the armatures is null.

Another quantity that is of interest is the capacity of the capacitor formed by the two armatures. For the resistive switch the expression of the capacity is:

$$C_{\text{res}} = \frac{\varepsilon_0 A}{g}, \tag{3.12}$$

and for the capacitive switch is:

$$C_{\text{cap}} = \frac{1}{\frac{g}{\varepsilon_0 A} + \frac{t_d}{\varepsilon_0 \varepsilon_r A}}, \tag{3.13}$$

the dependence of the capacitance with the applied voltage, until it reaches V_{pi}, is presented in Fig.3.10.

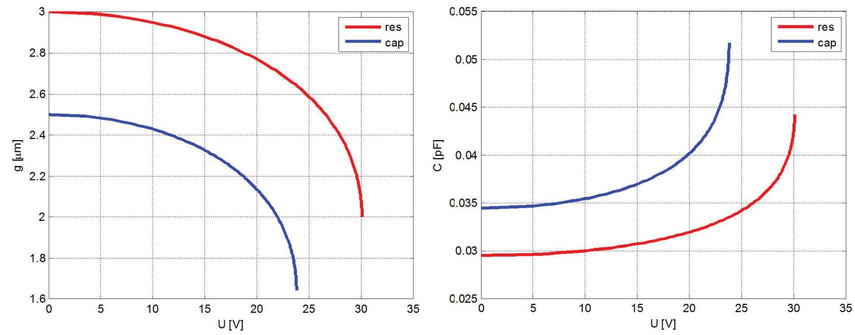

Figure 3.10: Dependency of the gap between the armatures (left) and capacitance (right) w.r.t. applied voltage.

In the case of the capacitive switch, the capacitance can be computed for voltages higher than V_{pi} (Fig.3.11), this being the capacitance of a capacitor with the distance between the armatures equal to t_d, filled with dielectric. Obviously this can be done for the resistive switch, its capacitance in actuated position will be infinite.

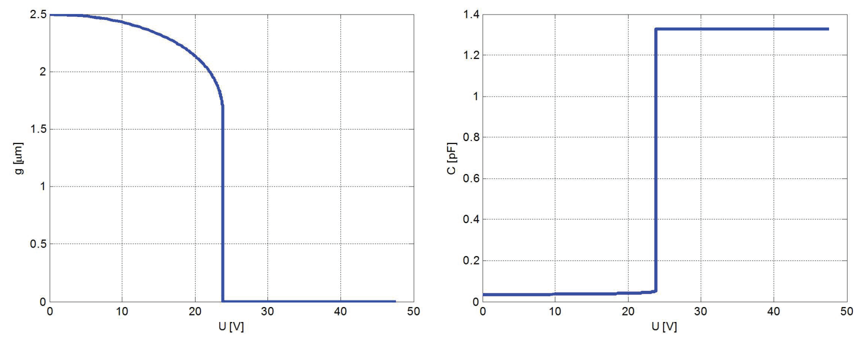

Figure 3.11: Dependency of the gap between the armatures (left) and capacitance (right) w.r.t. applied voltage that passes V_{pi}.

For this representation the the C-V curve aspect in the stable zone can not be distinguished because the capacity in down position is between 30 to 40 times higher then the capacity in up position. For this example the computed capacitance are: $C_{\text{down}} = 1.326pF, C_{\text{up}} = 0.034pF, C_{\text{pi}} = 0.051pF, C_{\text{down}}/C_{\text{up}} = 38.5, C_{\text{down}}/C_{\text{pi}} = 25.6$.

If we imagine now that the voltage slowly decreases, the phenomena are not reversible because now the problem is different, we have different initial conditions. We now have a capacitor which has only dielectric between the armatures. A capacitor in down position, with only dielectric, requires a lower voltage than V_{pi} to be maintained in this state.

The condition is that the elastic force corresponding to the stretching of the spring with g_0 distance, force that is directed up, to be lower than the corresponding electrostatic force formed between the armatures, without any air gap at which the weight of the armature is added, both oriented in down direction.

The condition mentioned to keep the switch actuated is:

$$V \geq \frac{t_d}{\varepsilon_r}\sqrt{\frac{2}{\varepsilon_0 A}(kg_0 - G)}. \tag{3.14}$$

Obviously, if the weight is greater than the corresponding elastic force of g_0 elongation, there is no need to have a voltage applied between the armatures to keep the switch actuated. Moreover, this kind of switch will never be not actuated.

To estimate the mass of the considered numerical example we suppose that the armatures are made from aluminum, which has a mass density of 2.7g/cm^3, and assume a $0.5\mu m$ thickness of the armature, resulting a gravity force of $1.3 \cdot 10^{-10}$ N, against the elastic force of the spring corresponding to the maximum elongation which is $kg_0 = 3 \cdot 10^{-5}$ N. Consequently, the mass can be neglected.

The minimum value of this expression (3.14) is called the pull-out voltage and has the value:

$$V_{\text{po}} \geq \frac{t_d}{\varepsilon_r}\sqrt{\frac{2}{\varepsilon_0 A}kg_0}. \tag{3.15}$$

If the applied voltage becomes smaller then V_{po} the upper armature is released, the representation of the actuation and deactuation processes for the dependence of the gap between the armatures and the capacitance with the applied voltage is presented in Fig.3.12.

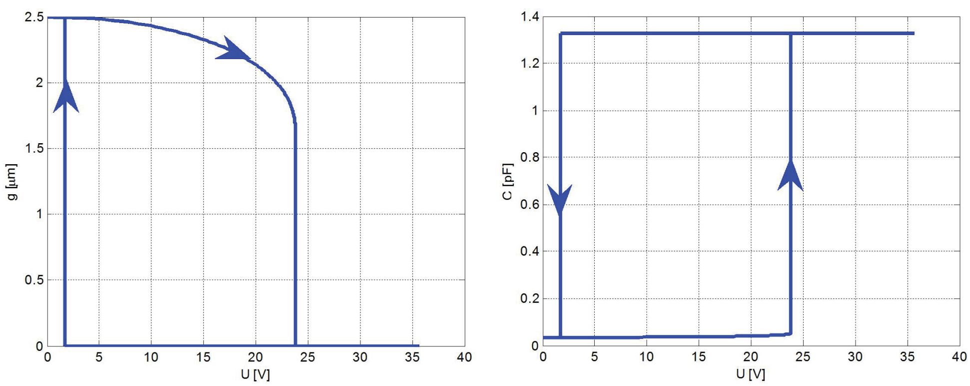

Figure 3.12: Dependency of the gap between the armatures (left) and capacitance (right) w.r.t. applied voltage.

3.2.2 Numerical Modelling in Static Regime

Numerical modeling consists of conceiving an algorithm dedicated to solving the model equations. What is interesting is that every issue taken separately is linear in terms of constitutive relations (material). In electrostatic domain, materials are linear and in mechanic domain the spring is linear (constant k is not dependent on the force applied). However the coupling between the domains is nonlinear, so even this simple problem is nonlinear.

Using MATLAB software the problem can be solved simply by writing a code to solve the nonlinear equation (3.7).

Using Spice software the circuit that describes (3.7) is shown in Fig.3.13, where *B1* and *B2* are behavioral sources. *G* is a voltage controlled current source. The element *R1* does not have any significance.

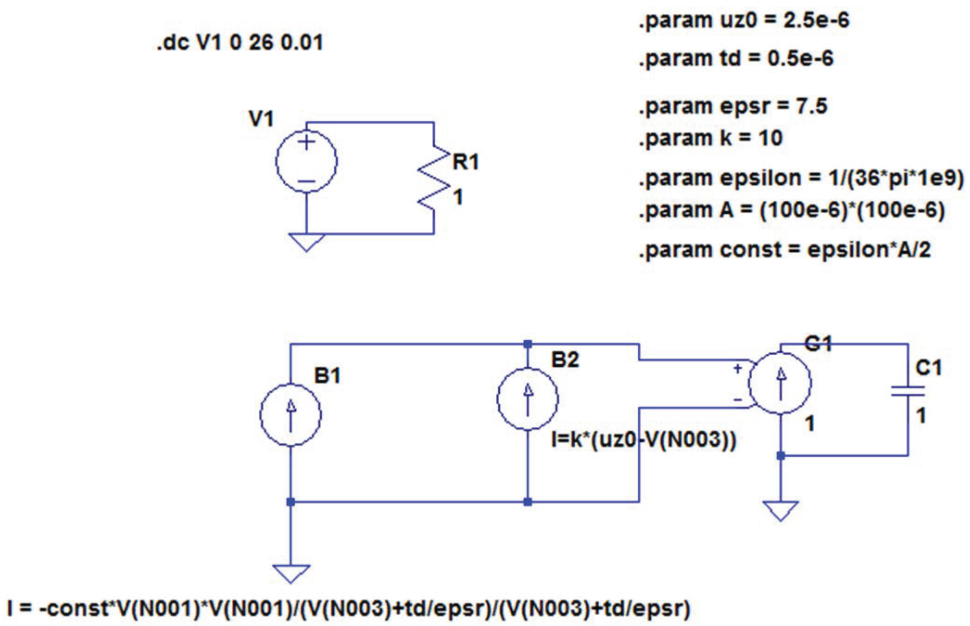

Figure 3.13: Spice circuit used to solve the nonlinear equation.

In Fig.3.14 the dependency of the displacement w.r.t. the applied voltage obtained by simulating the circuit is presented. To do the validation, the values computed in MATLAB are compared with the values obtained by simulating the circuit in Table 3.1. The small difference between the data validates the models.

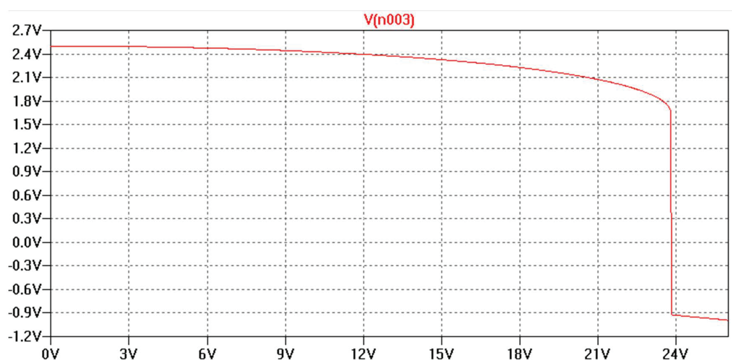

Figure 3.14: Dependency of the displacement with applied voltage obtained from SPICE.

Table 3.1: Comparison between values computed in MATLAB and values obtained by simulating the SPICE circuit.

MATLAB Values		SPICE Values	
V = 23.8036	g = $1.644\cdot10^{-6}$	V = 23.8500	g = $1.660\cdot10^{-6}$
V = 23.1914	g = $1.858\cdot10^{-6}$	V = 23.1900	g = $1.858\cdot10^{-6}$
V = 21.0397	g = $2.072\cdot10^{-6}$	V = 21.0300	g = $2.072\cdot10^{-6}$
		V = 21.0400	g = $2.072\cdot10^{-6}$
V = 16.3650	g = $2.286\cdot10^{-6}$	V = 16.3399	g = $2.286\cdot10^{-6}$
		V = 16.3499	g = $2.286\cdot10^{-6}$
		V = 16.3599	g = $2.286\cdot10^{-6}$

3.3 Conclusions

This chapter presents the analytical and numerical modelling of the most simple 1D model than can be imagine for a RF-MEMS device, the one of a parallel plane capacitor with one armature suspended from a spring. The analysis starts from the most general case, the dynamic analysis, were electrostatic, structural mechanic and fluid flow (represented in this study by the damping force) are coupled together. As a particular case making $\mathrm{d}/\mathrm{d}t = 0$ the static analysis was performed. For both regimes the modelling steps set in the previous chapter were checked for two constructive types of switches, capacitive and resistive.

The analytic analysis was done by solving the nonlinear differential equation. For the numerical modelling the problem was studied in three ways. MATLAB by a script that solves the nonlinear differential equation, APPLAC using a model that is implemented in the program and SPICE creating a *netlist* circuit that also solves the nonlinear differential equation. The SPICE equivalent circuit is indeed a similar circuit, based on the electric-mechanic similitudes. It was concluded that, in the dynamic simulations, both inertial force and the damping force have to be considered and in the static regime only the electrostatic force influence the structure. The gravitational force is never relevant.

Parametric analysis was conducted upon the lumped quantities extracted from the solutions, studding the effective elastic (stiffness) constants k, k_s, damping (that depends on the quality factor Q) coefficient b and effective mass (resonant frequency $\omega_0 = 2\pi f_0$) m_{eff}.

The static simulation of the model (including the MATLAB, APLAC or SPICE equivalent circuit) allows the extraction of the pull-in voltage, while its dynamic simulation allows the extraction of the switching time. Even the analysis model is the simplest one, the simulation results are meaningful and very valuable for the designers, allowing a better quantitative understanding of the switch operation. Although more accurate results will be obtained below, by using more complex models, the qualitative understanding of the essential aspect of RF-MEMS switches operation may be easier reached by the analysis made in this chapter. The conclusions and knowledge acquired by this analysis became a skeleton for the next more detailed and more accurate models. The main idea is to improve the SPICE model, by using more accurate circuit parameters, without increasing its complexity or changing its topology.

Chapter 4

Multiphysics Modelling of MEMS Switches and their Validation

This section presents the multiphysics modelling of a MEMS switch. As study case, two switches (QIAN and IMT) will be analyzed, which differ by the geometrical configuration and the way that the contact between the membrane and transmission line is made. One has a capacitive contact (QIAN) and the other a resistive one (IMT). For both configuration will be computed the pull-down voltage and their RF behavior (S parameters at the RF ports) in their stable states, UP and DOWN.

For both MEMS switches the 2D and 3D numerical models were developed, and then they were simulated in the static and dynamic regimes by using Finite Element Method (FEM) within COMSOL Multiphysics software package, coupling the Electrostatic, Structural Mechanic and Fluid Dynamic phenomena that occur in the operation of these devices. The full wave electromagnetic field was analysed by using the Finite Integration Technique to solve Maxwell equations with electromagnetic circuit element (EMCE) boundary conditions.

The QIAN structure [7], presented in Fig.4.1 - Left, consists of a CPW made by two grounded conductors on which the membrane (bridge) is placed, suspended over a signal line, along which the RF signal is propagated, if the membrane is in the UP state. In the DOWN state of the membrane, the RF signal is blocked due to the high capacitance between the two conductive plates (which makes a short circuit to ground). The signal

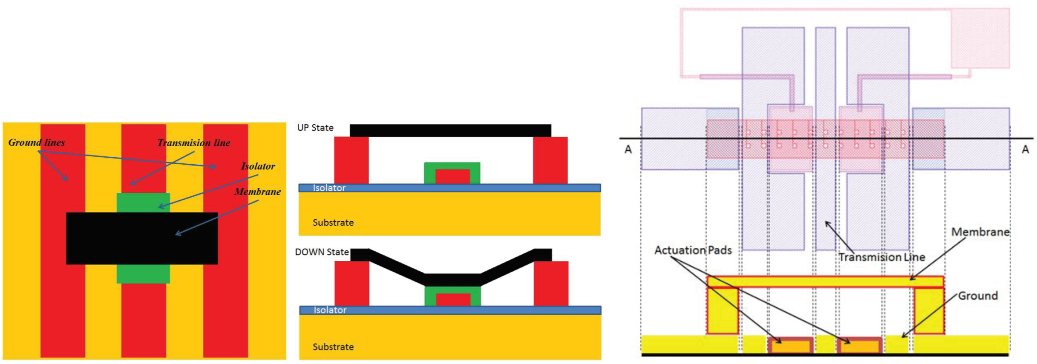

Figure 4.1: Left:Qian Structure (Left upper view of the geometry, Right transversal section, Up and Down stable positions) Right: IMT Structure. The transversal section is not to scale.

line is thinner in the middle; this part is serving as the actuation electrode having the same width as the signal line, coated with an isolator layer. The RF signal has a level much lower than those of actuation voltage, generating as consequence a neglected attraction force.

The IMT structure, presented in Fig.4.1 - Right which shows the practical realization of this switch, consisting of two grounded conductors on which the membrane is placed, suspended over a signal line, along which the RF signal is propagated if the membrane is in UP state. The construction with two actuation pads is preferred to the classical one which has the dielectric placed on the RF signal line, due to the fact that in this case the dielectric degradation problem that appears in down state is avoided [2].

4.1 2D Multiphysics Static Models of MEMS Switches

To reduce the computational effort, for beginning it is considered a two-dimensional, plan parallel model, in which the edge effects of the mobile membrane are neglected. Due to the symmetry with central, longitudinal ZOY vertical plane, the analysis is done only for a half of the structure (Fig.4.2). The transmission line, the insulating layer and the silicon substrate are excluded from the computational domain, being replaced by physical boundary conditions.

In the static regime the time variation is neglected ($d/dt = 0$) for any quantity and it is assumed that there is not any energy transfer (either electrostatic or elastostatic) i.e. top armature is in a equilibrium position, an immobile body. Therefore a pair of coupled electrostatic (ES) and structural mechanical (MEC) problems have to be solved.

Study Case: Static Analysis of the QIAN Structure

The parametrized geometric model of the QIAN structure is presented in Fig.4.2. Here are also indicated the materials of the components of the switch.

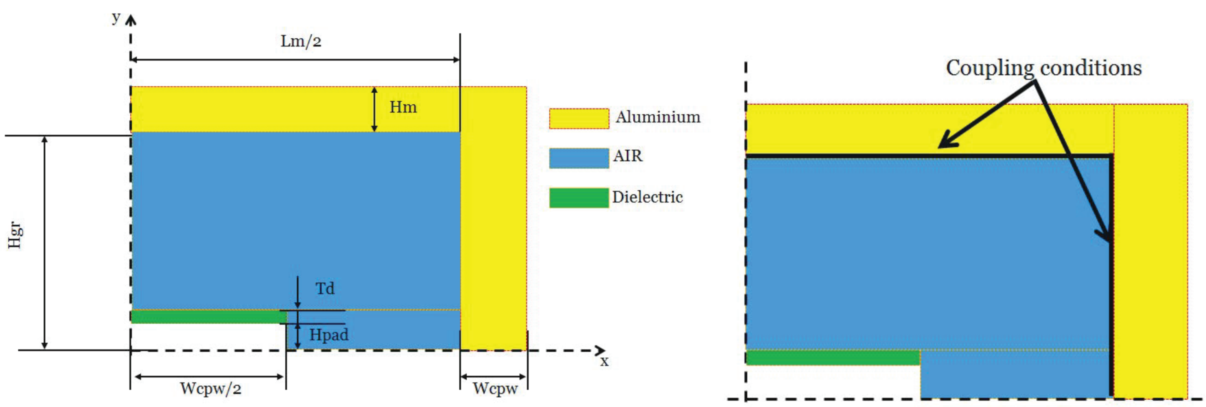

Figure 4.2: Qian Structure. Parametric model and coupling conditions.

Two static mutually coupled problems have been solved, one electrostatic and one elastostatic (Fig.4.3 - Left). To solve the multiphysics problem a nonlinear solver was used. The applied voltage U on the electrode was varied from 0 to 40 V. The pull-in voltage (V_{pi}) was assumed as the last value of the applied voltage U for which the procedure is convergent. The value $V_{pi} = 30.045$ V is obtained. The shape of the membrane at V_{pi} is presented in Fig.4.3 - Right that shows the color map of the displacements.

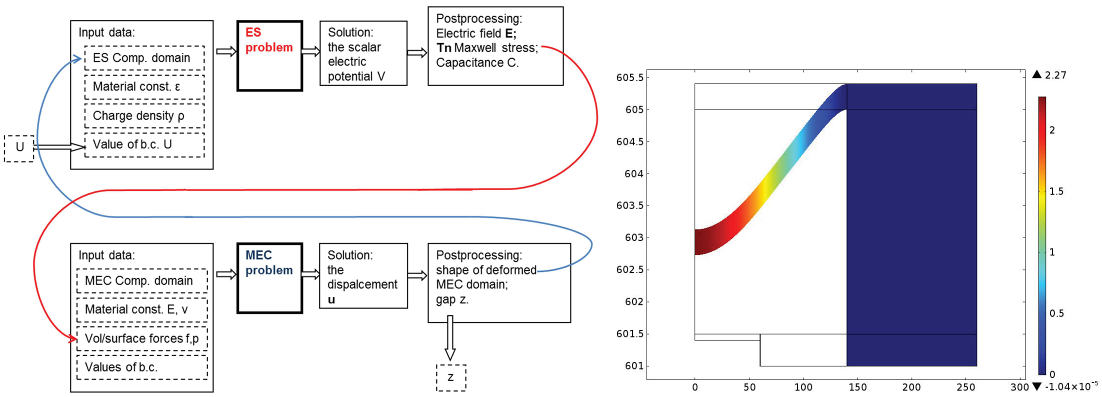

Figure 4.3: Left: The mutual coupling of the electrostatic (ES) and the structural-elastostatic (MEC) problems, Right:The shape of the membrane at V_{pi}.

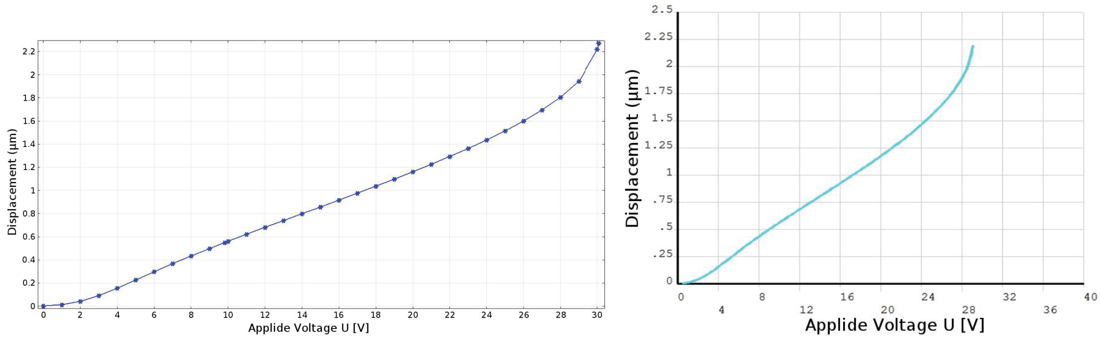

Figure 4.4: The central displacement z vs. applied voltage U. Right: COMSOL simulation, Left: ANSYS.

The variation of the membrane position vs. applied voltage is presented in Fig.4.4 - Left, the central displacement z for a point situated in the middle of the membrane on the bottom surface, were the maximum displacement should occur. The verification was done by comparing the simulation results obtained with COMSOL with results of ANSYS, which gives the result depicted in Fig.4.4 - Right. In this case, the last actuation voltage at which the algorithm converges is $U = 29.07$ V.

The relative difference of the latest pull-in voltage calculated by COMSOL and ANSYS is about 3%, which is quite acceptable. The maximum displacement of the membrane computed by COMSOL is $z_{\text{pi}} = 2.26\mu$m, and with ANSYS is $z_{\text{pi}} = 2.19\mu$m, having also a deviation below 3%.

Study Case: Static Analysis of the IMT Structure

The parametrized geometry of the IMT structure, is presented in Fig.4.5 - Left, where are identified the materials values of the the switchş components.

The shape of the membrane at V_{pi} is presented in Fig.4.5 - Right.

The applied voltage U on the electrode was changed from 0 to 10 V. The pull-in voltage (V_{pi}) was assumed to be the last value of the applied voltage U, for which the procedure is convergent, and it has the value $V_{\text{pi}} = 7.896$ V and the maximum displacement has the value $z_{\text{pi}} = 1.42\mu$m. The variation of the membrane position vs. applied voltage is presented in Fig.4.6, the central displacement z for a point situated in the middle of the membrane on the bottom surface, were the maximum displacement should occur.

Parametric 2D Multiphysics Analysis

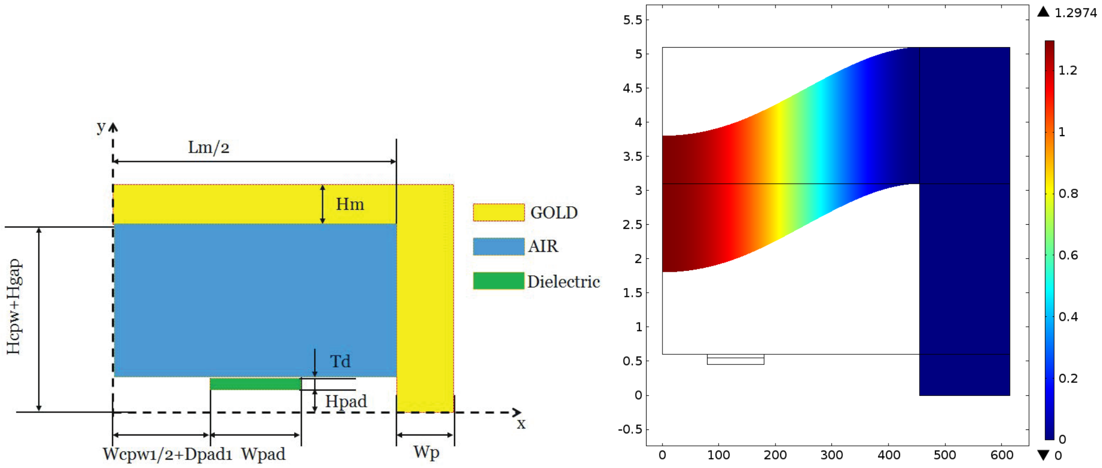

Figure 4.5: Left: Parametrized geometry of the IMT structure. Right: Displacement for the last stable position

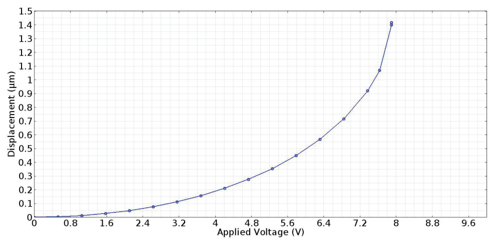

Figure 4.6: The central displacement z vs. applied voltage U (COMSOL simulation).

In this section the IMT switch (Fig.4.5) is analyzed from the point of view of designer, who is interested to understand how the pull-in voltage is influenced. A parametric study was carried out, considering several changes, such as dimensions of the actuation pads and the bridge length.

The width of the actuation pads W_{pad} was changed from 100μM to 300μm with a step of 50μm. The length of the membrane L_m was varied from 910μm to 1110μm with a step of 100μm.

An important aspect to consider is the residual stress σ. Ideally $\sigma = 0$, since in our case this stress is not desirable, however it may be present in the membrane, as a result of the fabrication process. The residual stress σ was varied from 0 MPa to 60 MPa with a step of 10 MPa.

The first set of simulations focused on the static pull-in voltage, for different configurations and different residual stresses σ in the membrane. In each run, the load characteristic curve was obtained, the instability of the nonlinear solver occurring at the pull-in voltage V_{pi}.

In order to validate the models, coupled MEC-ES models have been simulated both in COMSOL and in ANSYS. The models have identical grids and identical boundary conditions, but the formulations, as well as the used nonlinear solvers, are different. Even so, the relative differences in the resulted pull-in voltage is under 3%.

4.2 3D Multiphysics Models of MEMS Switches

To retrieve relevant quantities related to the switching action from one stable state to the other, the investigation of the dynamic behavior needs simulations that take into account several physical effects: electrostatic actuation, mechanical motion as well as the air damping [10], [3]. The fluid flow damping phenomena, which is essential for accurate computations of dynamic characterisitics (e.g. commutation time, pull-out voltage, etc.) can be simulated only by using 3D models. An accurate dynamic behavior can be extracted only if three field problems are coupled as in Fig.4.7.

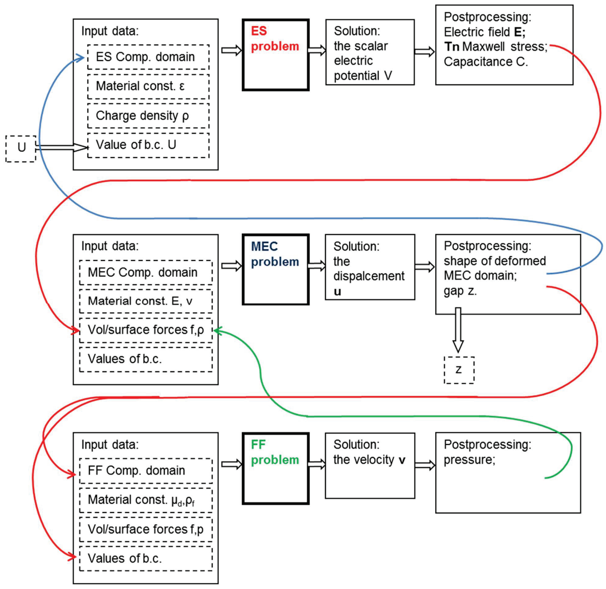

Figure 4.7: The mutual coupling of the electrostatic (ES), the dynamic-structural (MEC) and Fluid Flow (FF) problems

This section is dedicated to the dynamic, transient analysis. The evolution in time of the membrane deformation is simulated, aiming to compute the dynamic pull-in voltage, which is the lowest voltage that can be applied so that the switch is actuated (the membrane crashes on the actuation pad). From the transient response, may be extracted also the actuation time as well as the capacitance variation in time, including the corresponding capacities of the two stable states UP and DOWN.

4.2.1 Study Case: Dynamic Analysis of the QIAN Structure

Due to the structural symmetry, only a quarter of the geometry is modeled. The signal line, the ground lines and the actuation pads are not relevant for the multiphysics models that focus on the switching between the UP and DOWN positions, when there is no signal passing through the signal lines. The simulation flow starts with a static simulation aiming to compute the static pull-in voltage $V_{\rm pi}$. Fig.4.8 holds the obtained solution.

In order to extract the switching duration, a dynamic analysis have to be done. The time evolution of the maxim displacements of the membrane, for several constant actuation voltages are represented in Fig.4.9 - Left. As we expected, without considering

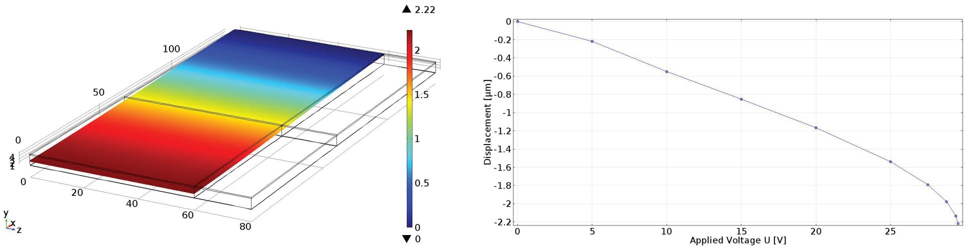

Figure 4.8: Left: Displacement of the bridge at pull-in voltage, Right: Displacement of the bridge middle point w.r.t. applied voltage U.

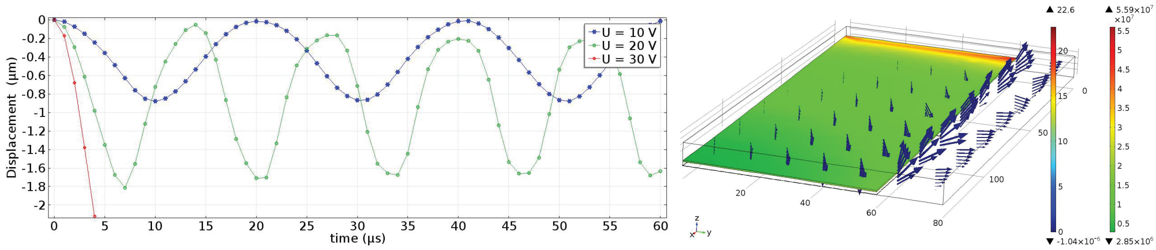

Figure 4.9: Left: Maxim displacement of the bridge in time, for different applied voltage U, Right: Typical solution of the ES+MEC+FSI problem.

air damping, the oscillations are continuous undamped. The dynamic pull-in voltage is $V_{\mathrm{dpi}} = 25.8$ V and the actuation time is 7μs.

The relative difference between the static and dynamic pull-in voltage is reaching almost 13% which is quite high, meaning that Fluid-Structure-Interaction (FSI) has to be taken in consideration, in order to model the dumped effect of the air during the transient regime.

Fig.4.9 - Right contains a typical solution of the ES+MEC+FSI problem is presented for an applied voltage of 30 V.

Considering air damping, which correspond to a model closer to the reality, the found dynamic pull-in voltage is around 31 V (Fig.4.10). The relative difference between static and dynamic analysis with air-dumping is around 3%, which is quite acceptable. Comparing the Fig.4.9 - Left and Fig.4.10 it is obvious the fundamental importance of the air dumping in the MEMS modelling.

In conclusion for a more accurate extraction of the dynamic pull-in voltage, the air damping has to be taking in consideration. It can not be neglected in any conditions.

A series of difficulties experienced in the modelling with COMSOL package. Using

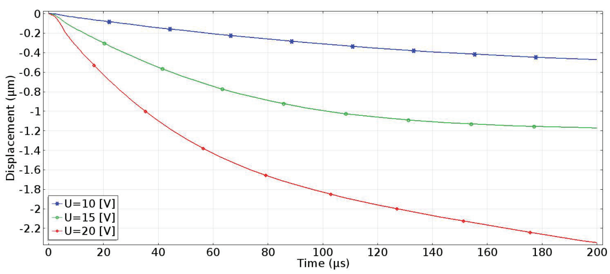

Figure 4.10: Evolution in time of the displacement of the bridge for different applied voltages with air damping.

COMSOL default settings, the simulations fail and to obtain reliable results, attention must be paid to the model building and to the solver settings. When dynamic analysis is conducted, consideration should be given to the time stepping method.

4.2.2 Contact Modelling

In order to model the full dynamic response of the movement of the switchş membrane (to simulate a compete switching cycle UP-DOWN-UP), the contact force must be included in the numerical model. The Penalty/Barrier Method is used. This method combines elements of the penalty procedure with elements from the barrier method.

The model of the QIAN structure was excited by a smooth step voltage of 35 V. The transient analysis was performed for 40 μs, considering the contact force.

Without taking in consideration the air damping, is obtained the time variation of the displacement of the bridge center presented in Fig.4.11- Right. Fig.4.11 - Left, holds the shape of the bridge that is in contact with the isolator layer, for a quarter of the model.

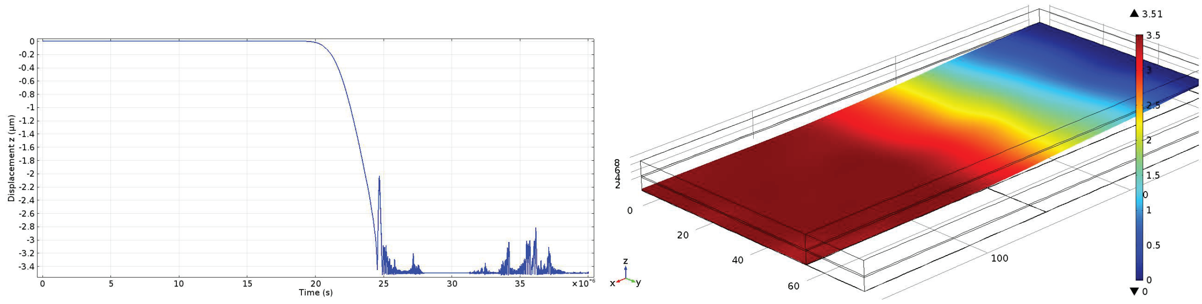

Figure 4.11: Left: Time dependence of the displacement of a point from the center of the bridge, Right: Shape of the membrane at contact with the isolator layer.

Now it is applied to the same model a voltage signal having a rectangular pulse shape with a rise and fall of 5 μs (Fig.4.12 - Right). Without taking in consideration the air damping, the time dependence of the displacement of the bridge center, resulted from transient simulation is presented in Fig.4.12 - Left. The analysis was performed for 60 μs. Unrealistic oscillations may be observed when the bridge is released.

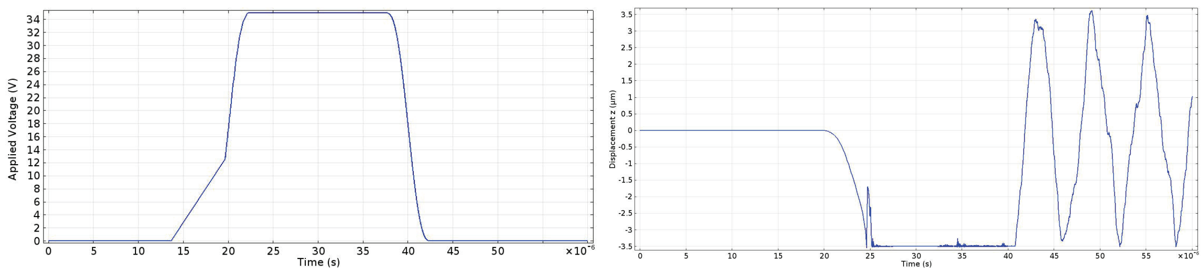

Figure 4.12: Left: Time dependence of the Applied Voltage. Right: Time dependence of the displacement of the bridge center.

Including the air dumping, the time dependence of the displacement of the bridge center, resulted from the transient simulation under step excitation is presented in Fig.4.13 - Right. In Fig.4.13 - Left is presented the time dependence of the displacement of the bridge center for the applied voltage of rectangular signal type. As we expected the switching time is clearly increased.

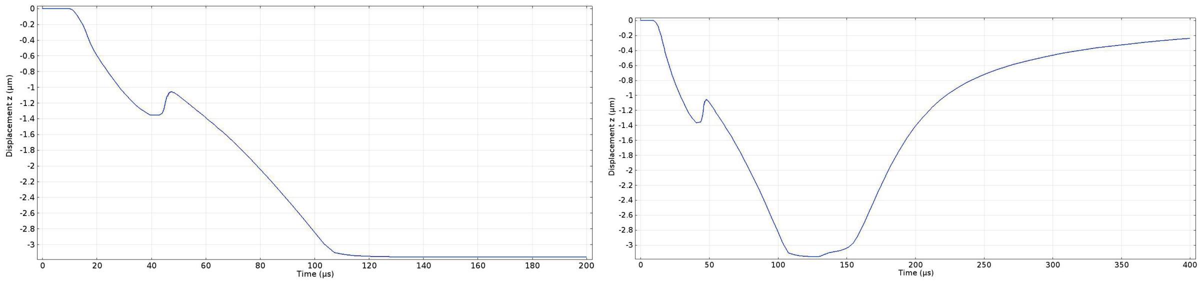

Figure 4.13: Left: Time dependence of the displacement of the bridge center for a step input signal, Right: Time dependence of the displacement of the bridge center for a 20μs pulse signal.

4.2.3 Influence of Membraneş Perforations

In order to facilitate the extraction of the sacrificial layer, during their technological process, the MEMS membranes have perforations. From structural point of view, the existence of holes has the advantage that some of the residual stress in the bridge are reduced, thus reducing the Youngs modulus of the RF-MEMS structure. Another advantage of membrane perforation is that it can reduced the pull-in voltage and increase the switching speed due to the easier fluid flow.

In this section it will be analyzed the influence of membrane perforations on the dynamic behavior of the membrane, taking in consideration the air damping. In the membrane 3 perforations have been placed, having the area of 400 μm^2. The model represents only a quarter of the switch, thus the full model will contain 12 perforations in the membrane.

Figure 4.14 - Left contains a typical solution of the ES+MEC+FSI problem, presented for an applied voltage of 25 V. The dynamic pull-in voltage is found around 26.5 V. Adding perforation in the membrane caused a drop of V_{dpi} with approximately 15%. The time dependence of the position of the bridge center, excited by an applied voltage of 25 V, is presented in Fig.4.14 - Right. We noticed a smoother monotonic movement.

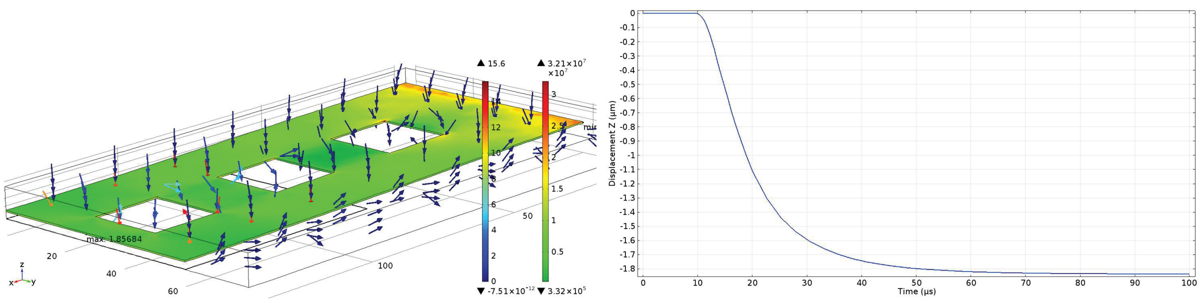

Figure 4.14: Right: Typical solution of the ES+MEC+FSI problem for an applied voltage of 25 V, Left: Time dependence of the bridge center position, when the applied voltage is 25 V.

4.2.4 Parametric 3D Multiphysics Modelling

Modelling of MEMS devices is a challenging task due to the coupling of multiphysics phenomena that have to be considered. In these conditions, parametric modelling is even more challenging and it is a compulsory step before the optimization. Some papers such as [7] are referred to this challenge, but only the RF behavior is investigated. In this current section is investigated also its multiphysics parametric behavior.

In the case of the QIAN structure, there are three parameters that may vary: the width of the central line of the CPW l_{cpw} was varied from 60μm to 200μm with a step of 40μm, the length of the membrane L_m was varied from 280μm to 580μm with a step of 100μm and the width of the membrane W_m was varied from 60μm to 120μm with a step of 20μm. The influence of the discretization was also investigated.

In conclusions, the results of parametric analysis show that the static pull-in voltage depends on the switch dimensions and material characteristics, decreasing with the length of the membrane and width of the electrode, while the maximum displacement remains approximately constant. For a more accurate extraction of the dynamic pull-in voltage the air damping has to be taking in consideration. The switching time is also linked to the geometric parameters, decreasing with the increasing in the length of the membrane, otherwise the corresponding capacities of the two stable states UP and DOWN remains approximately constant.

4.3 3D Electromagnetic Modelling of MEMS Switches

Radio frequency micro-electro-mechanical switches (RF-MEMS) are miniature devices that use mechanical movement to change the configuration of a radio frequency circuit. The simplest devices are used to perform a short circuit or an open circuit in a transmission line. The RF-MEMS switch analyzed in this section is the QIAN and IMT2.5.

Between the membrane and central line there is a space, which creates a capacity, the membrane being placed transversely to the line. When it is not actuated, the value of this capacity is quite small, so that the RF signal that crosses the line is not disrupted. When a sufficient high voltage is applied to the actuation electrode placed under the membrane, the switch is actuated and the RF signal is blocked, because the capacitance between the signal line and the ground became high.

Study Case: the QIAN Structure

RF simulation of this device was performed by using of the *chamy* package, which is a in house software toolbox dedicated to the computational modeling of the passive high-frequency integrated circuit components and their interaction with the electromagnetic environment, aiming to calculate the frequency characteristics of the examined devices.

Once the problem was defined, by using the input file, the program generates state matrices of the modeled device using Finite Integrals tTechniques (FIT) to discretize the Maxwellş equations in the full wave electromagnetic regime with Electromagnetic Circuit Elements (EMCE) type boundary conditions.

The parametric geometric model of QIAN structure is composed of three metal lines, made from aluminum (ALUM), placed on an buffer layer, made from silicon dioxide (SiO_2), deposited on a silicon (Si) substrate. The transmission line is thinned in the middle part, where a dielectric layer of silicon nitride (Si_3N_4) is deposited. The membrane, made also from from aluminum, resting on the two ground lines is suspended above the center line.

For the UP state of the RF-MEMS device, Fig.4.15 and Fig.4.16 show the amplitude and phase of S_{11} respective S_{21} parameters, obtained by EM simulation with *chamy*.

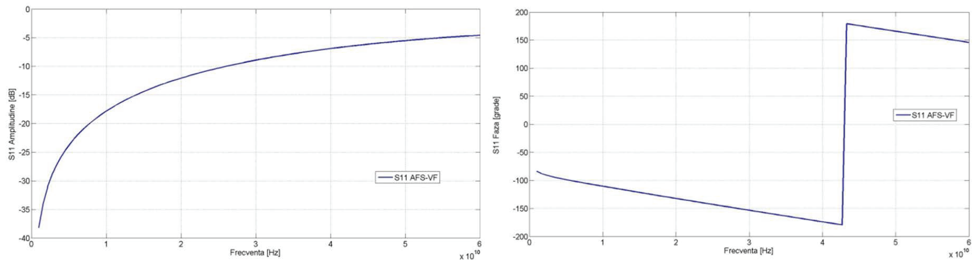

Figure 4.15: S_{11} Parameter for Up position. Amplitude and phase.

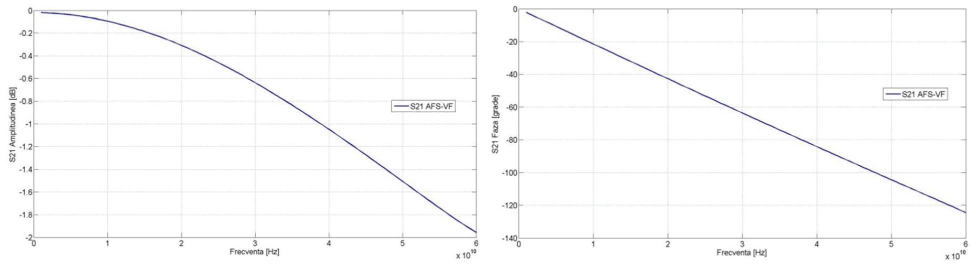

Figure 4.16: S_{21} Parameter for Up position. Amplitude and phase.

For the DOWN state of the RF-MEMS device, Fig.4.17 and Fig.4.18 shows the amplitude and phase of S_{11} respective S_{21} parameters, obtained by EM simulation with *chamy*.

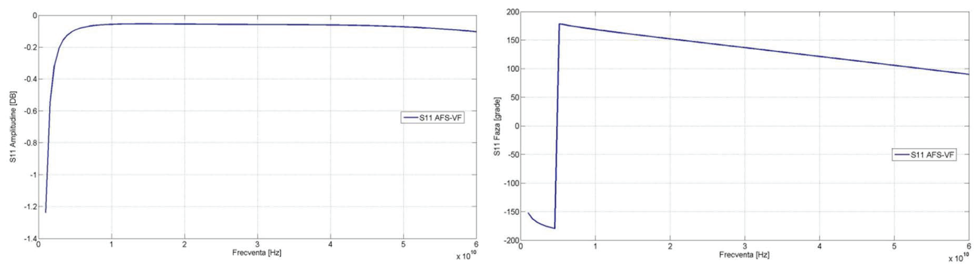

Figure 4.17: S_{11} Parameter for Down position. Amplitude and phase.

To validate the results obtained with *chamy*, a comparison was made with the results from the literature [7]. Comparing the results, one can observe a good agreement of results obtained by *chamy* software with the results presented in [7] obtained with a commercial software and, which was experimentally validated (less than 4 dB error). Consequently, the numerical model proposed in this section is validated.

To observe the influence on the frequency response of the width of the membrane, a parametric analysis was conducted, varying it in the range of 60 μm to 180 μm.

The influence of membrane perforations on the RF characteristic expressed by means of S parameters was also analyzed. Different types and configurations of the perforations applied to the membrane are investigated, both from the S parameters point of view and the computational time, needed for each simulation.

In conclusion, this study shows that there is no need to take the perforations of the membrane into account in the RF models of capacitive MEMS switches, because the

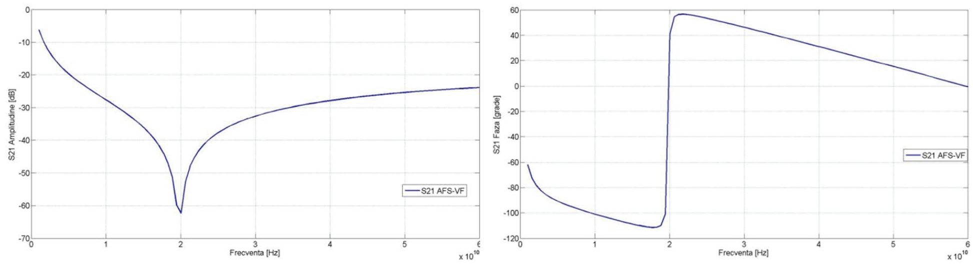

Figure 4.18: S_{21} Parameter for Down position. Amplitude and phase.

relative variation of the simulation results of the RF models with and without holes is around 0.04 % both in the UP and DOWN states. This is a great advantage for the RF model since it allows the meshing of the membrane with a coarser grid. This leads to an important decrease of the computational resources: the model has a lower order (in our example 3 times lower) and the computational effort decreases both in pre-processing (matrix generation times is about 10 times lower) and in solving (5-6 times lower), while the error is acceptable.

Study Case: the IMT2.5 Structure

It is composed of three metal lines, from gold (GOLD), placed on a titanium layer (TI), deposited on a silicon (Si) substrate. The actuation of the device is made by two action pads, made from gold, placed on each side of the signal line (middle line) coated by a silicon oxide (SiO_2) layer. The membrane, also from gold, resting on the two ground lines, is suspended above the central line. Similar to the QIAN case study, the S parameters were extracted for the UP and DOWN position. An additional study was made for the lines alone in order to extract the p.u.l parameters.

4.4 Experimental Characterization and Validation

The practical realization of the structurs and its RF characterization was done at National Institute for Research and Development in Microtechnologies (IMT) - Bucharest, Romania (`www.imt.ro`).

Two wafers of high resistivity silicon were processed and several types of structures were added:

- CPW lines;
- RF-MEMS switches with a construction of bridge type for which the following parameters were varied: width of the bridge w_{Wm} (80 and 200 μm), length of the bridge l_{Wm} (910, 1110 and 1210 μm), width of the actuation pad w_{act_pad} (200 and 300 μm);
- RF-MEMS switches with a construction of cantilever type for which the following parameters were varied: length of the cantilever (910, 1110 and 1210 μm), width of the actuation pad (100, 200 and 300 μm);

The Fig.4.19 - Right presents a photography of the IMT2.5 structure, by optical microscope, Fig.4.19 - Left of the IMT structure, Fig.4.20 - Right of the QIAN structure and Fig.4.20 - Right of the CPW lines. The relative difference of the RF response from the wafers is approximately 10%.

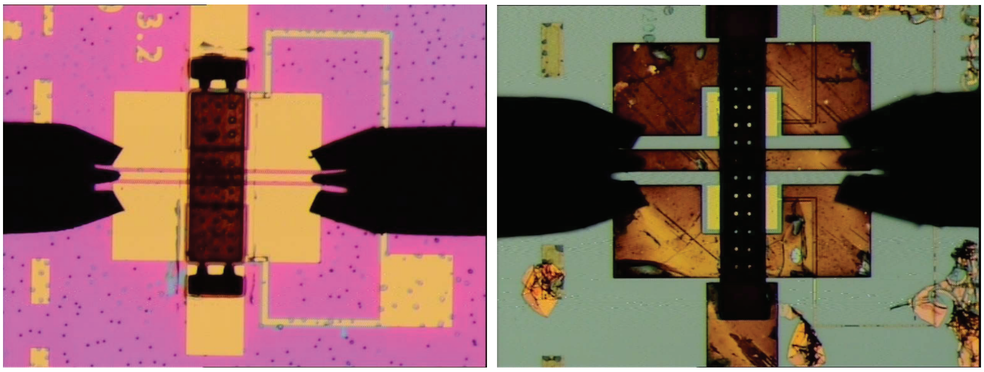

Figure 4.19: Right: Microscope view of the IMT2.5 Structure., Left: Microscope view of the IMT Structure.

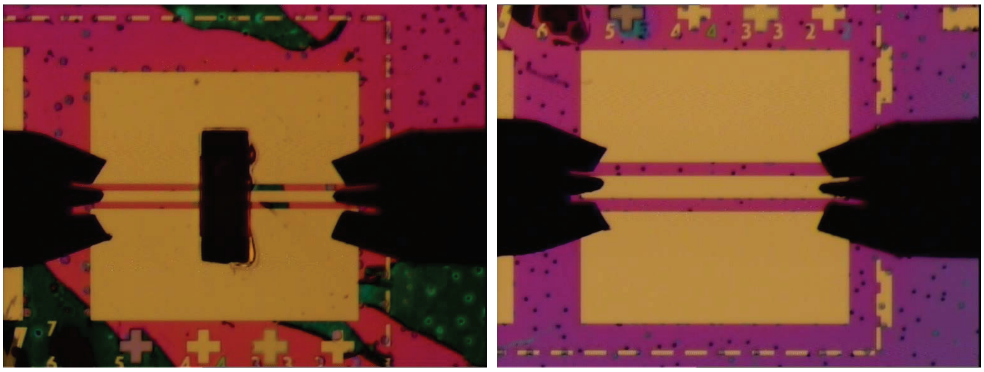

Figure 4.20: Right: Microscope view of the QIAN Structure, Left: Microscope view of the CPW lines.

The validation of the numerical model is done by comparison of the simulation results with the results of the experimental measurements. Fig.4.21 contains this comparison presenting S_{11} and S_{21} for wafer 1.

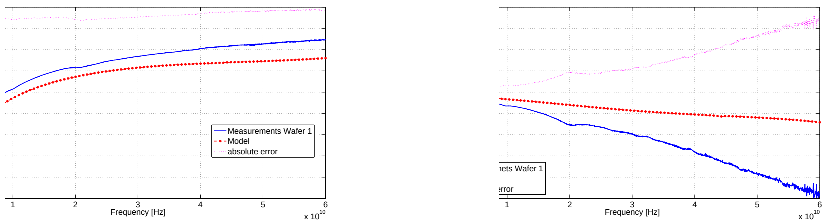

Figure 4.21: S_{11} and S_{21} parameter for Up position. Comparison between the computed RF response and measured RF response from wafer 1.

One can notice a good agreement between numerical results and measurements, thus validating the numerical model.

4.5 Conclusions

This chapter is dedicated to the development of numerical multiphysics and RF models for RF-MEMS study cases. Static, dynamic and EM analysis were performed, in order to compute the field-solution of these three problems. Electric and magnetic problems (ES and RF) are strongly coupled with mechanical problems (MEC - Structural and FD - Fluid dynamics). The verification of the results was made by using several software packages (COMSOL and ANSYS based on FEM for multiphysics; *chamy* based on FIT and HFSS based on FEM for RF numerical modelling). The validation of the developed numerical models is done by experimental means. The experimental results describing the RF behavior of two RF-MEMS switches are obtained from literature and from measurement made by IMT.

After the static analysis of the QIAN structure the difference between numerical solutions, computed by COMSOL and ANSYS is about 3%. The maximum displacement of the membrane computed by COMSOL is $z_{pi} = 2.26 \mu m$. The difference under 3% proves the correctness of the numerical solving procedure. From the parametric study, it can be noticed that the pull-down voltage decrease with the increase of the pad width and length of the membrane, being very sensitive to the residual stresses in the membrane. The 2D models are enough accurate for the static analysis, consequently they are suitable for computing of the static pull-in voltage.

For a more accurate extraction of the dynamic pull-in voltage, the air damping has to be take in consideration. The relative difference between the static and dynamic pull-in voltage decreasing in this manner from 13% to 3%. In order to model the air dumping, a 3D model is necessary. Consequently, in order to compute the switching time, a dynamic, 3D multiphysics analysis (ES-MEC-FSI, strongly coupled) have to be done. To simulate a switching cycle (OFF-ON-OFF) a contact model must be included.

From RF point of view, comparing the results, one can observe a good agreement between our results obtained using *chamy* software and the results presented in [7], obtained with a commercial software and validated by measurements. As consequence, the numerical method proposed in this thesis is validated.

To obtain reliable simulation results, many aspects must be considered. Each field problem (ES, MEC, FF, RF, contact) have to be very careful described to the computer (the geometry, material constants, field sources, and boundary conditions), the couplings between problems and the solving methods, as well as their parameters (appropriate meshing, linear and nonlinear solvers, time integration).

The study conducted on the effects of membrane perforations upon the RF characteristics shows that there is no need to take the perforations of the membrane into account in the electromagnetic modeling of capacitive MEMS switches, because the relative error between the models with and without holes is around 0.4 % both in the UP and DOWN states. This is a great advantage for the RF modelling since it allows the meshing of the membrane with a coarser grid.

The human and computational effort to simulate in an accurate manner the mechanical and RF behavior of RF-MEMS switches, based on their numerical field models is quite substantial. Therefore for an efficient (re)design of these devices, another approach should be used. This is why, in this chapter an important attention was dedicated to the parametric modelling and in the next chapter the model reduction will be addressed.

Chapter 5

Reduced Models for RF MEMS Switches

The modeling of MEMS devices is an important issue in their design loop [3, 10], that has to provide eventually a compact model, described as a circuit netlist. This will allow the designer to easily simulate the whole system in which the switch is embedded.

5.1 Extraction of Lumped Parameters

The switchş behavior is characterized by several parameters. The compact models mentioned above (as that in Fig.5.1) need equivalent (effective) lumped parameters that characterize their components. The transversal variable capacitor is an essential component for understanding the switch behavior.

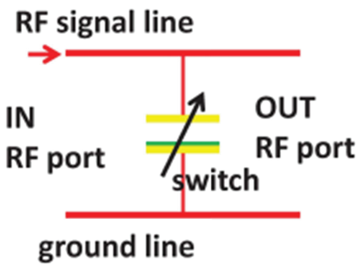

Figure 5.1: Conceptual macro-model of a RF MEMS switch

5.1.1 Lumped Parameters of the Multiphysics Model

The main idea of this chapter is to consider the 1D MEMS model as a compact - reduced model of the more accurate 3D model. That means we have to extract the characteristic lumped parameters of 1D model from the simulation results of the 3D model. For example, starting from the solutions of the 3D simulations, one can identify equivalent ("effective" values for the effective stiffness, mass and damping coefficients. The effective stiffness k_{eff} may be extracted also from simulations of 2D model. Below is presented a complete and coherent procedure to extract the multiphysics reduced model and its SPICE equivalent circuit based on the accurate 3D multiphysics and RF-EM simulations of the modeled device.

In order to extract the effective elastic coefficients, a coupled MEC+ES multiphysics model is analyzed in static regime. To obtain a reliable result, the model must includes a proper coupled field formulation and it should be analyzed with appropriate numerical simulation techniques. To illustrate this aspects, QIAN and IMT test structures are analyzed. Several attempts to extract the effective elastic coefficient were tried, one based on analytical formula and four based on numerical computations that use the results given by the coupled model described in the previous section.

In conclusion, we may say that the nonlinear (cubic) approximation of the elastic force is more appropriate, at least for the QIAN study case.

The effective mass of the bridge may be extracted from the results of dynamic analysis performed for the coupled 3D electrostatic-structural problem. Applying a voltage less than V_{pi}, un-damped oscillations of the bridges displacement was obtained.

The effective mass is expected to be lower than the real mass of the membrane, because the several inside points have lower oscillations than the central point of the membrane.

Extraction of Damping Coefficient

Because the dumping is produced mainly by the air resistance, the damping coefficient may be extracted from coupled 3D electrostatic-structural-fluid flow problem solved numerically by FEM. Applying a voltage less than V_{pi} a damped displacement of the bridges was obtained. The membrane movement is monotonic, totally damped, without any oscillation.

The effective damping coefficient is computed by a first order least square approximation of the damping force dependence with respect to the velocity.

The Multiphysics Reduction Procedure

In conclusion, the procedure of extraction of the effective m, b and k coefficients, consists by the following 10 steps:

- Step 1. Do coupled static numerical ES+MEC simulations (e.g with FEM) for increasing values of the actuation voltage V_0. Record position z and electrostatic energy W_{ES}.
- Step 2. Compute the dependence of the switch capacitance $C(z) = 2W_{ES}/V_0^2$ on the membrane displacement and approximate the capacitance inverse $1/C(z)$ with $\alpha z + \beta$ by affine regression (find coefficient α and β by the least square method).
- Step 3. Compute the dependence of the electrostatic force $F_{ES}(z)$ vs. displacement z by applying the generalized force theorem $F_{ES}(z) = V_0^2/2 \cdot dC(z)/dz$.
- Step 4. Do a cubic least square approximation of the dependence found at step 3 in order to find k and k_s.
- Step 5. Do a coupled dynamic transient structural-electrostatic analysis (e.g. by FEM), under a step voltage very less than the pull-in voltage. Extract the resonance frequency from the un-damped obtained oscillation of the membrane, by FFT. Compute the effective mass as $m = k/\omega^2$.
- Step 6. Do a coupled transient analysis (structural-electrostatic-fluid flow) for a step actuation voltage. Record position $z(t)$ and electrostatic energy $W_{ES}(u)$.

- Step 7. Apply again the parameter identification for the second order linear system, by computing the dependence of the capacitance with respect to the position $C(z)$ and approximate it with an expression $1/(c_1 z + c_2)$. Based on it, compute the electrostatic force $F_{ES}(z)$ by using the generalized force theorem.
- Step 8. Compute the velocity $v = dz/dt$ and the acceleration $a = dv/dt$.
- Step 9. Compute damping force $F_d = F_{ES} - F_{el} - F_{in}$, where F_{el} uses the effective stiffness coefficients and found at step 4 and $F_{in} = m \cdot a$ uses the effective mass found at step 5 and the acceleration computed at step 8.
- Step 10. Do a first order least square approximation of the damping force dependence with respect to the velocity and compute the effective damping coefficient b.

The complexity reduction procedure of the nonlinear multiphysics RF-MEMS switches model is based not only on mathematical principles but rather on some essential physical principles. This makes the reduced model to preserve more essential characteristics of the real device.

Lumped Parameters of the Radio Frequency Model

The conceptual model of RF MEMS switch contains two transmission lines and a transversal dipolar microwave circuit element. These structure might be approximated by a compact circuit with lumped parameters, but more accurate results may be obtained if we adopt a hybrid approach, combining lumped with distributed elements. The two lines are modeled as 1D transmission lines while for the transversal dipole, an equivalent circuits with lumped parameters is considered . The simplest choice is to model the transversal element as a RLC series circuit with lumped parameters (having values dependent w.r.t. z- the membrane position, as a continuous function, or at least known for the two stable position UP and DOWN.

TL-lumped RF compact models of MEMS switches can be extracted with a robust procedure from the numerical solution of Maxwell equations with ECE boundary conditions. To to do it, the simulations are made in three cases: lines alone, UP and DOWN positions.

As a consequence, the proposed algorithm dedicated to extraction of the TL-lumped of the RF compact modelş parameters has 4 steps:

- Step 1. Browsing frequency, do simulations of the line alone (e.g. by numerical solving of Maxwelş equations) and extract the frequency dependence lineş S parameters.
- Step 2. Do simulation of the Full Wave EM field in the MEMS device in the UP position.
- Step 3. Improve the line parameters using the average value of C_{up} over the frequency range.
- Step 4. Do simulation for the DOWN position. Extract Y_{down} compute R_{mem}, C_{down} and L_{mem}.

Using closed form relationships has the benefit to do a fast fitting and correction of parameters of the compact model. The proposed algorithm has the advantage that it may incorporate easily the frequency dependence of parameters, either for the TLs or for the switch inductivity.

5.2 Validation of the Reduction Procedure

Multiphysics Compact Model

To validate the method of extracting the effective elastic coefficient, the nonlinear equation in z is solved for several applied voltages and the instability point is recorded. The solving was done in SPICE. In this respect, an equivalent (actually based on mechanic-electric similitude) circuit that synthesizes these equations can be easily built and simulated for an increasing of values of the actuation voltage. The schematic of this circuit is shown in Fig.5.2. It contains behavioral current sources, the currents representing forces. The actuation voltage is modeled by the independent voltage source V_1, the ES force "flows" through the behavioral current source B_1 and the elastic force is modeled by the behavioral source B_2. The values are scaled for a more robust numerical analysis.

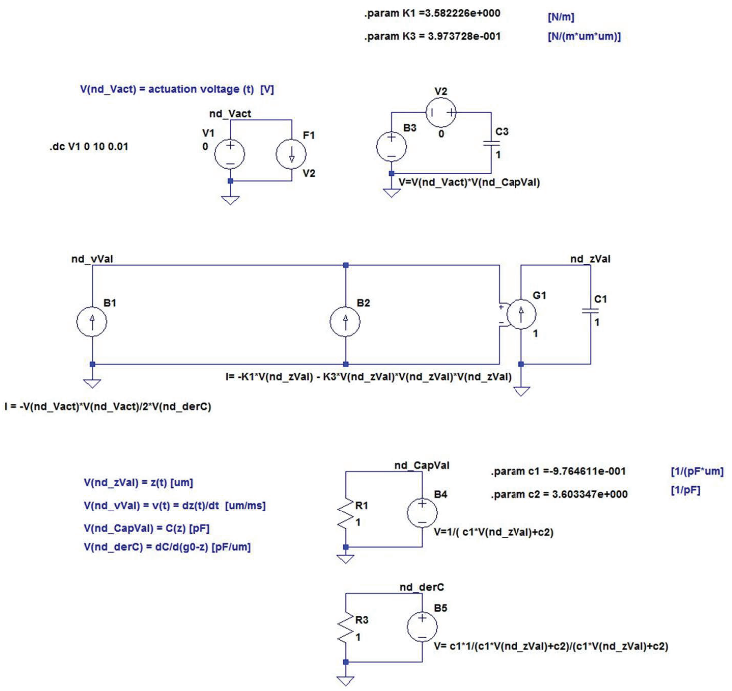

Figure 5.2: Static macro-model: Spice circuit that models several relationships that include the global mechanical end electrostatic phenomena in the coupled static simulation. (Here: $k_1 = k, k_3 = k_s, c_1 = \alpha, c_2 = \beta$).

For the IMT structure the relative error of the pull-in voltage is computed with respect to the reference value of 7.89 V obtained from the static FEM simulation. A better image is obtained if displacement-voltage curves are compared for increasing values of the actuation voltage, as in Fig.5.3. It can be noticed that the averaging approaches behave worse than the analytical approach, and the cubic least square is able to recover not only a very accurate pull-in voltage (relative error less than 0.5 %), but the whole behavior of the displacement curve.

To extract the effective mass and damping coefficient, a 3D numerical model has to be analyzed. The extraction method is tested on a simple benchmark (Fig.5.4), a cantilever switch, actuated by a voltage that is applied between it and a ground that ensures an initial airgap of g_0. The cantilever is of parallelepiped shape, with length $l = 300\mu m$, width $w = 20\mu$, and thickness $t = 2\mu m$.

The SPICE circuit that describes the macro-model is shown in Fig.5.5. The actuation

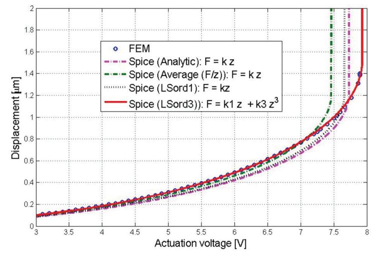

Figure 5.3: Displacement-actuation curves: reference (FEM) vs. SPICE macro-model simulations with values of effective coefficients obtained from various methods.

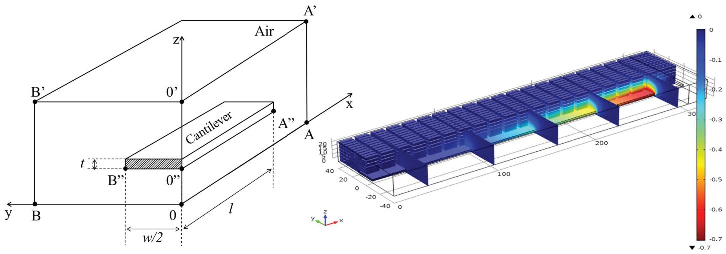

Figure 5.4: Left: Benchmark problem: the computational domain includes half of the cantilever. (Size of domain: $320\mu m \times 15\mu m \times 8\mu m$. Drawing is not at scale). Right: typical distribution of the electrostatic field norm.

voltage is modeled by the independent voltage source V_1. The ES force "flows" through the behavioral current source B_1. The elastic force is modeled by the behavioral source B_2. The effective mass is a capacitance and the "current" flowing through it is the inertial force. The displacement, is the voltage at the node labeled "nd_zVal".

The effective elastic coefficient is extracted in the same way as for the 2D model. The results are: $k = 23[N/m], k_1 = 0.27[N/m], k_3 = -0.084[N/m(\mu m)^2], c_1 = -6[(pF \cdot \mu m)^{-1}], c_2 = 30[(pF)^{-1}]$.

By transient simulation of ES-MEC coupled problems under small applied voltage, an oscillating frequency $f_0 = 25[kHz]$ is obtained, resulting an effective mass of $m = 9.6 \cdot 10^{-6}[Kg]$. Even if only linear model was used to extract the mass, the macro-model response is better when it implements the cubic dependence of the elastic force.

The final validation is carried out for a three field coupled problem electrostatic-structural-fluid flow. As a result of the damping force - velocity fitting obtained the value of the dumped coefficient $b = 2.710^{-6}[Kg/s]$. This value is used as a conductance in the SPICE equivalent model circuit (Fig.5.5 - Right). The dynamic answer of the macro-model as well as the reference answer obtained from the field simulation are shown in Fig.5.5 - Left. For the step excitation of $5V$ the relative error is 0.19 %, while for an excitation of $6V$ step the relative error is 0.35%.

The deviation of the behavior of compact model from the reference FEM-field model is quite acceptable, validating the extraction procedure of the SPICE multiphysics model presented above.

Compact RF Model

The compact model is inspired by the geometry of the device and it consists of a transmission line that model the RF signal line and an equivalent lumped parameter circuit model for the switch itself. In the UP position the lumped parameter model of the switch is merely a capacitance, whereas in the DOWN state position, the lumped

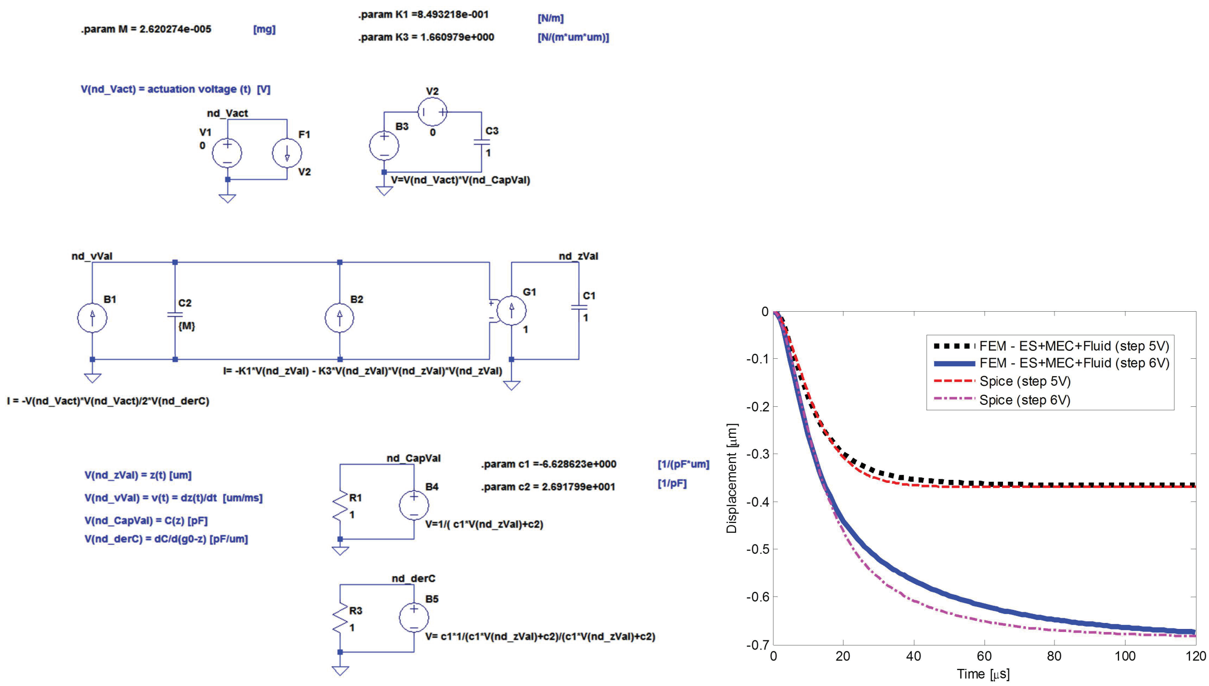

Figure 5.5: Right: Multiphysics Macro-model; scaled values are used. Left: Dynamic answer of the macro-model and reference (FEM results).

parameter model is a R, L, C series connection. The values of the RF compact model are derived from the RF simulations, and therefore it is important to have accurate RF models of the device. In order to extract the lumped parameters of switch itself, a kind of de-imbedding, must be done, in order to eliminate the effects of the input and output connection lines.

Two test configurations were studied, QIAN and IMT structures, and acceptable relative errors in the S-parameters were obtained. Improvements are possible if a frequency dependent model is adopted for the switchş inductivity.

The QIAN benchmark is useful since [7] provides also measurements for the S parameters in the UP and DOWN positions, and thus, any RF model we use to illustrate the proposed extraction algorithm is validated.

The proposed algorithm was illustrated for a shunt switch but no conceptual difficulties are foreseen for series (inline) switches. For the QIAN structure, the modeling of the frequency dependence for the line parameters is not so important, but the modeling of the frequency dependence for the switch inductivity is relevant, the relative error in the S parameters decreasing from 8 % to 2.5 %. For the IMT Structure, which is a resistive one, the compact model is good but not as accurate as the QIAN results.

Compact Parametric RF Model

The method proposed in the previous section extracts the TL macro-model solely from three simulated frequency responses: one for the signal line alone, one for the switch in the passing through state and one for the switch in the blocking state. The characteristic parameters of the TL-lumped model are extracted by fitting. It is based on closed form expressions derived from the transmission lines theory, and it has the advantage that very few steps are required to obtain a relatively accurate RF macro-model. Moreover, the method allows naturally the inclusion of frequency dependence behavior, either in the transmission line parameters, or in the switch inductivity.

This section is an expanded version of the above method, in which the extraction procedure is modified so that a parametrized TL-lumped macro-model be obtained. To do this, not only the frequency response is computed from an RF simulation, but also its sensitivity with respect to the parameter that varies, which is consequently used to compute the sensitivity of the lumped components of the macro-model.

In this section two test problems are studied: QIAN structure, which is a capacitive switch of bridge type, placed transversely with respect to the RF signal line, and another configuration, IMT2.5 structure, which is a resistive switch, with two actuation pads.

A MIMO distributed system with $(n-1)$ inputs and $(n-1)$ outputs may be properly defined to characterize the behavior of a device with n terminals. If a terminal is excited by current (voltage), then its voltage (current) is the output signal. Therefore, according to its excitation, the ECE is described by a frequency dependent, hybrid matrix. In particular, impedance $\underline{\mathbf{Z}}$ or admittance $\underline{\mathbf{Y}}$ matrices may be properly defined and extracted from the field solution.

To assembly the discrete state space system of a switch configuration, we use the Finite Integration Technique (FIT) to discretize the Maxwell equations.

In conclusion the parametric TL-lumped RF macro-models for MEMS switches can be extracted with a robust procedure from the solution of Maxwell equations with electromagnetic circuit element (ECE) boundary conditions in three cases: lines alone, UP and DOWN positions.

The main idea is based on the use of closed form relationships which allow a natural de-embedding of the RF results in order to compute the switch admittance, which is eventually fitted either with a capacitance in the UP position, or with a RL or RLC series connection in the DOWN position. Two fitting possibilities have been tested: one based on vector fitting and the other based on a simple least square approach. In some cases, the obtained values might not reflect physical phenomena, but for the capacitive switch tested, both methods predicted accurately the resonance frequency and its sensitivity.

If sensitivities of the port quantities are computed simultaneously with the EM solution, then they can be used to evaluate the sensitivities of the switch admittance and, consequently, the sensitivities of the lumped parameters. Thus, only RF computations for the nominal value of the design parameter are carried out, thus avoiding the need to re-mesh the problem for other values of the design parameters.

The proposed algorithm was illustrated for shunt switches but no conceptual difficulties are foreseen for series (inline) switches. The algorithm may easily incorporate the frequency dependence of line parameters if needed.

The parametric macro-models are very accurate (error less than 3 %) with respect to reference macro-models extracted from independent EM simulations.

5.3 Mixed Domain Coupled Macro-models

In order to describe the entire behavior of the switch, the complete compact model has to include both RF model and multiphysics- movement model (we will call it as a mixed macro-model). The RF signal lines are best described by transmission lines (TL) models with 1D distributed parameters, whereas for the switching part the components with RLC lumped parameters are used. The parameters extraction as it was presented before is based on the results obtained from three RF simulations carried at device-level model, with the finite integration technique, and use of a fitting procedure based on closed form relationships for the TL-lumped macro-model.

The SPICE equivalent circuit of mixed-coupled macro-model is obtained by simply replacing the switch capacitance in the RF schematic by a model that connects it with the multiphysics compact model, as in Fig.5.7. A fixed capacitance has been added in parallel with the parametric one. It corresponds to the electric field lines that close through the substrate, and it has been computed by a separate electrostatic problem for the substrate. The validation of the model built so far is done by comparing the RF results (S parameters) of the mixed schematic with the results from the EM simulation (Fig.5.6). A relative error of 2.5% is obtained.

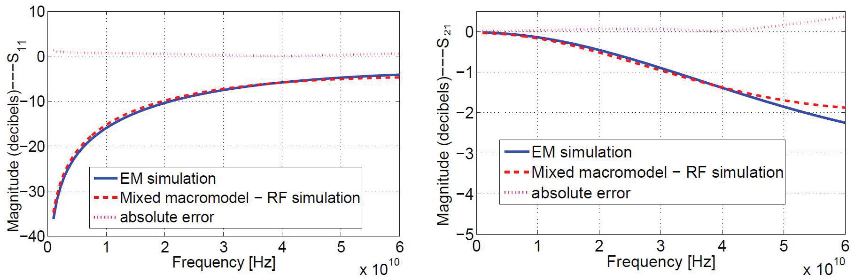

Figure 5.6: EM simulation vs. mixed macro-model RF simulation: left - return loss (S_{11} signal pass), right - insertion loss (S_{21}, signal pass).

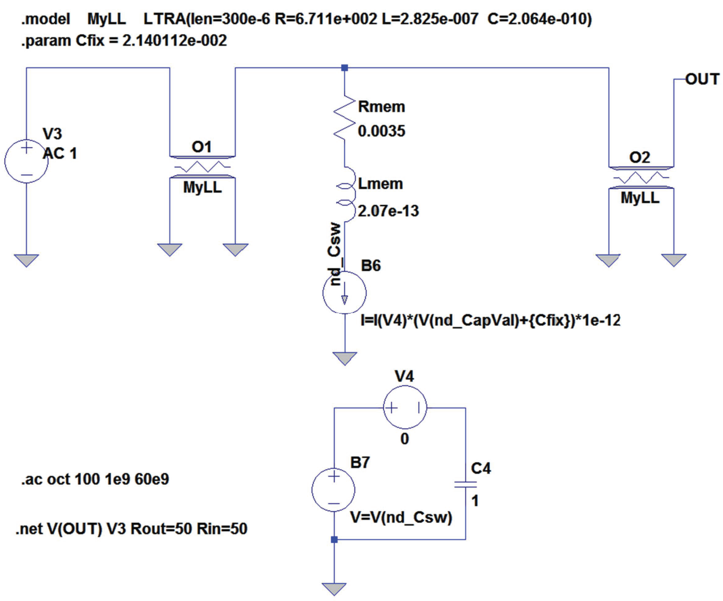

Figure 5.7: Mixed macro-model: the RF part, the switch model is a current source controlled by the capacitance value that is taken from the multiphysics part (voltage at node "n_CapVal" in the multiphysics part of the schematic).

The mixed macro-model of a RF-MEMS switch, with few degrees of freedom is extracted from several (FEM-FIT) numerical analysis of the device with distributed parameters (solving structural, fluid flow, electrostatic and full wave EM fields). All extracted parameters are combined into a single *netlist* model (circuit implemented in Spice), which is controlled by the MEMS actuation voltage and is excited with the RF signals. This model comprising two coupled circuits (one describing the movement is depicted in Fig.5.5

and other is the RF circuit from Fig.5.7) allows fast and accurate simulations of electro-mechanical and RF behavior of RF-MEMS switches under several excitations. These two circuits are mutually coupled by means of controlled sources. A relative error less than 3% in the S parameters and less than 1% in the pull-in voltage is obtained, which is very satisfactory given the low order imposed for the reduced model. This model is able to describe also the dynamics of mechanical movement, and if the a model for the contact is added, it is able to model the switching time also.

5.4 Conclusions

In this chapter a procedure with 14 steps for extraction of the effective m, b and k multiphysics coefficients and the RF compact modelş parameters was developed and it is illustrated and validated for two study-case.

For the extraction of the effective elastic coefficient the averaging approaches behave worse than the analytical approach, and the cubic least square is able to recover not only a very accurate pull-in voltage (relative error less than 0.5 %), but the whole behavior of the displacement curve.

By solving a three field coupled problem electrostatic-structural-fluid flow and by fitting of the relation the damping force - velocity is obtained the dumped coefficient is b = 2.710^6[Kg/s]. The dynamic answer of the macro-model as well as the reference answer obtained from the field simulation for the excitation of 5V step used do derive the lumped parameters (relative error is 0.19 %), as well as for an excitation of 6V step (error is 0.35%).

The frequency simulation of the RF compact model for the QIAN benchmark in the down position reveals an error of 8.1 % in the case of constant inductivity for the switch and 2.5 % when a frequency dependent inductivity is used. In the case of the IMT benchmark, there is no improvement if a frequency dependent inductivity is used, the relative error is 7.1 %.

A special attention is given to the parametric models of RF MEMS switches and their realization in SPICE. At the end of this chapter a mixed macro-model of a RF-MEMS switch, with few degrees of freedom is extracted from several (FEM-FIT) numerical analysis of the device with distributed parameters (solving structural, fluid flow, electrostatic and full wave EM fields). All extracted parameters are combined into a single *netlist* model (circuit implemented in Spice), which is controlled by the MEMS actuation voltage and is excited with the RF signals. This model comprising two coupled circuits (one describing the movement is depicted in Fig.5.5 and other is the RF circuit from Fig.5.7) allows fast and accurate simulations of electro-mechanical and RF behavior of RF-MEMS switches under several excitations. A relative error less than 3% in the S parameters and less than 1% in the pull-in voltage is obtained, which is very satisfactory given the low order imposed for the reduced model.

The developed model was validated for the RF behavior point of view, by experimental means and by comparisons of SPICE simulation results with field simulations done by *chamy*/FIT, COMSOL/FEM, CST (in both frequency domain FIT-FD and time domain -FIT-TD) and HFSS/FEM.

Chapter 6

Final conclusions and original contributions

RF-MEMS switches are very small mechanical devices built on semiconductor chips having the size less than one millimeter and therefore they are measured in micrometers that use mechanical movement to create a short circuit or an open circuit of the RF signal in the transmission lines. The growing demand for micro-fabricates having a superior energy performance, determined the reorientation of manufacturers to integrated circuits and hence the scientific community to the technology of micro-electro-mechanical systems, whose benefits, particularly in radio frequencies were highlighted in the last decade.

Modelling a device or a real system is an fundamental action of science and engineering, which consists in developing a series of abstract scientific and mathematical images and representations of the modeled object, or digital type-numeric representation of that object. Usually, precise models are more complicated and their extraction and simulation have a higher cost. Choosing the most appropriate model depends on the context and it reflects a compromise between the optimum accuracy and simplicity which are contradictory requirements. A parametric model describes the entire class of real, similar objects. The number of geometric parameter and/or material constants which allows the identification of the instance-object in the class defines the complexity of the parametric model.

Modelling of MEMS devices is an important issue, both theoretically and practically. This is because the modelling and simulation of these devices should take in consideration more physic phenomena, it is necessary, finally, a coupled modeling, more complicated, of multiphysics nature. Otherwise, the nowadays technology of integrated circuits production is unthinkable without a prior modelling, simulation and optimization of the new designed devices.

The design of this micro-electro-mechanical devices that operate at radio frequencies requires effective procedures for modelling and software capable of simulating coupled multiphysics models involving mechanical, electrical, thermic or fluid flow phenomena. Research in the field of RF-MEMS is to develop at a cost as low as possible effective models with high accuracy, but to have a reduced complexity. Simulation and optimization of these models using computing machines, leads to the development of designs for MEMS devices with the best performance.

After the introduction, the main objectives of the thesis were presented and the actuality and importance of the thesis subject was highlighted, in the first chapter the current state micro-electro-mechanical systems modelling was presented, the chapter contains a summary of the general concepts, then talks about the current state and the importance of the subjects also making a critical study of the publications from the recent years and prior research in this domain.

The second chapter is dedicated to a brief presentation of the theoretical concepts, knowledge and background on which the entire work is developed. In this chapter the open problems are formulated and the suitable research methodology is identified.

In the third chapter are presented the analytical and numerical modelling of the most simple 1D model which can be imagine for a RF-MEMS device, the similar of a parallel plane capacitor with one armature suspended from a spring. The fourth chapter focuses on the creation of numerical multiphysics and RF modelling. Static, dynamic and EM analysis were performed in order to compute the field-solution of the problems. The validation of the results was made using several software packages.

The fifth chapter presents the mixed macro-model of a RF-MEMS switch extracted from several (FEM-FIT) numerical analysis of the device with distributed parameters (solving structural, fluid flow, electrostatic and full wave EM fields) together with an effective 14 steps procedure of extraction of the effective m, b and k multiphysics coefficients and the RF compact modelş parameters is presented. All extracted parameters are combined into a single *netlist* model (circuit implemented in SPICE), which is controlled by the MEMS actuation voltage and is excited with the RF signals.

6.1 Original Contributions

1. Critical presentation a State of the Art in Modelling of RF-MEMS Devices;
2. The analysiss based on analytic-approximation of the simplest 1D model for RF MEMS switches. Development of 2D and 3D models for RF-MEMS device. The multiphysics models based on coupled Electrostatic, Structural and Fluid Flow phenomena were simulated and validated;
3. An efficient algorithm for extracting a compact model with multiphysics lumped parameters, validate by comparison with field solution;
4. An efficient algorithm for extracting the lumped parameters for new compact reduced RF model for RF MEMS switches, validated with experimental and computational means;
5. A hybrid Parametric Mixed Domain Macro-models of RF-MEMS switches that includes both a model for the RF signal lines and a multiphysics model for electromechanical behavior;

These contributions were made by teamwork, especially in the ToMeMS research project coordinated by prof. Gabriela Ciuprina within LMN. As results from the list of 15 papers published by the author of the thesis, (first author on 4 of them) the original contributions are shared by the members of the research teams.

6.2 Future Research and Development

1. Development of new efficient modeling methodologies based on High Performance Computing Techniques;
2. Problem formulation, modelling, simulation and analysis of coupled Electrostatic - Structural - Fluid Flow - Contact - RF phenomena for accurate MEMS 3D models;
3. Experimental characterization of RF MEMS for computational models validation;
4. Efficient extraction of parametrized MEMS macro-models and their including it into optimization loops;
5. Efficient optimization of RF-MEMS devices for their optimal design;
6. Development of new MEMS for bio-applications;
7. Publication of the original results.

6.3 Dissemination of the Results

1. Gabriela Ciuprina, **Aurel Sorin Lup** and Alina Tomescu, “Parametrii S în Aplicaţii de Înaltă Frecvenţă”, Lucrările Simpozionului Naţional de Electrotehnică Teoretică, SNET2012, ISSN 2067-4147 (online), 6 pages.
2. Andreea Alexandru, **Sorin Lup** and Bogdan Diţă, “GDS2M Preprocessing Tool for MEMS Devices” Proceedings of the 8th International Symposium on Advanced Topics in Electrical Engineering, ATEE2013, May 23-25, 2013, Bucharest, Romania, 4 pages, ISBN 978-1-4673-5978-8, Pages: 1 - 4, DOI: 10.1109 / ATEE.2013.656345, WOS: 000332928500105. This paper was awarder 3^{rd} place in the IEEE student paper competition.
3. **Aurel-Sorin Lup** and Gabriela Ciuprina, “Analysis of membrane perforations on the RF behavior of capacitive MEMS switches”, Acta Electrotehnica, ISSN 1841-3323 vol. 54, no.5, pp. 252-255, 2013. - CNCSIS categoria B+ BDI VINITII (Russia), DOAJ (Sweden).
4. Gabriela Ciuprina, **Aurel-Sorin Lup**, Bogdan Diţă, Daniel Ioan, Ştefan Sorohan, Dragoş Isvoranu and Sebastian Kula “Mixed Domain Macro-models for RF MEMS Capacitive Switches”, The 10th International Conference on Scientific Computing in Electrical Engineering, SCEE 2014, July 22 - July 25, 2014, Wuppertal, Germany, SCEE 2014 Proceedings, 8 pages (ISI-WOS).
5. **Aurel-Sorin Lup**, Gabriela Ciuprina and Stefan Sorohan, “Parametric Multiphysics Static Models for a Bridge Type MEMS Capacitive Switch”, Conference Proceedings of 49th Universitiesower Engineering Conference - UPEC2014, 02-05/09/2014, Cluj-Napoca, Romania, Pages: 1-4, DOI:10.1109/UPEC.2014.6934635.
6. Sebastian Kula and **Aurel-Sorin Lup**, “Electrical schematics for 1d analysis of a bridge type MEMS capacitive switch”, Computer Applications in Electrical Engineering, volume 12, pages 407-421, ISSN 1508-4248, ISBN 978-83-7775-352-1.
7. **Aurel-Sorin LUP** and Gabriela Ciuprina, “Extraction of effective elastic coefficient from a coupled structural electrostatic simulation of a MEMS switch”, Inter-

national Symposium on Fundamentals of Electrical Engineering 2014, ISFEE2014, NOVEMBER 28-29, 2014, Bucharest, Romania, Pages: 1 - 5, DOI: 10.1109/ISFEE.2014.7050539.

8. Ruxandra Bărbulescu, **Aurel-Sorin Lup**, Gabriela Ciuprina, Daniel Ioan and A. Egemen Yilmaz, "Intelligent Particle Swarm Optimization of Superconducting Magnetic Energy Storage devices", International Symposium on Fundamentals of Electrical Engineering 2014, ISFEE2014, NOVEMBER 28-29, 2014, Bucharest, Romania, Pages: 1 - 4, DOI: 10.1109/ISFEE.2014.7050607.

9. Cosmin-Bogdan Diţă, **Aurel-Sorin Lup** and Daniel Ioan, "Reduced order RF macromodel for a MEMS capacitive switch", The 9th International Symposium on Advanced Topics in Electrical Engineering, may 7-9, 2015, Bucharest, Romania, Pages: 436 - 441, IEEE Xplore, DOI: 10.1109/ATEE.2015.7133844, (ISI).

10. Mihai Popescu, **Aurel-Sorin Lup** and Daniel Ioan, "Data structures for coupled structural electrostatic modeling of MEMS Switch", The 9th International Symposium on Advanced Topics in Electrical Engineering, may 7-9, 2015 Bucharest, Romania, Pages: 448 - 451, IEEE Xplore, DOI: 10.1109/ATEE.2015.7133846, (ISI).

11. Gabriela Ciuprina, **Aurel-Sorin Lup**, Daniel Ioan, Dragoş Isvoranu and Ştefan Sorohan, "Combined Multiphysics and RF Macromodels for Electrostatic Actuated Micro-Electro-Mechanical Switches", Conf. on the Computation of Electromagnetic Fields in Montreal from 28 June - 2 July, COMPUMAG 2015.

12. Gabriela Ciuprina, Cosmin-Bogdan Diţă, **Aurel-Sorin Lup**, Daniel Ioan and Alexandra Ştefanescu, "Extraction of TL-lumped RF macromodels for MEMS switches", IEEE MTTS International Conference on Numerical Electromagnetic and Multiphysics Modeling and Optimization for RF, Microwave and Terahertz Applications, Aug. 11-14, 2015, Ottawa, Canada, NEMO2015.

13. **Aurel-Sorin Lup**, Gabriela Ciuprina, Mihai Popescu and Daniel Ioan, "Parametric Multiphysics 3D Modelling of a Bridge Type Mems Capacitive Switch", The 17th International Symposium on Electromagnetic Fields in Mechatronics, Electrical and Electronic Engineering - ISEF015, 10-12 September, 2015, Conference Proceedings ISBN: 978-84-606-9102-0.

14. Mihai Popescu, **Aurel-Sorin Lup**, Ruxandra Bărbulescu, Gabriela Ciuprina and Daniel Ioan, "Using Object Oriented Data Structures for Optimizing MEMS Devices on Parallel Computers", The 17th International Symposium on Electromagnetic Fields in Mechatronics, Electrical and Electronic Engineering - ISEF015, 10-12 September, 2015, Conference Proceedings ISBN: 978-84-606-9102-0.

15. Mihai Popescu, **Aurel-Sorin Lup** and Ruxandra Bărbulescu, "Professional Package Software for Multiphysics Modelling. Application to MEMS devices", Scientific Bulletin, Series C Electrical Engineering and Computer Science, 2015.

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