On a MOS Computational Circuit

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***Abstract*—An analog MOS circuit performing the maximum element selection (WTA) is considered. The dependence of its behavior on the input list density is rigourously studied.**

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I. Introduction

The circuit under study was presented by Lazzaro in 1989 in his thesis, [1],[2]. Many improvements and applications of it have been issued since then, [3],[4]. However there hasn't been any thorough theoretical study of the circuit so far.

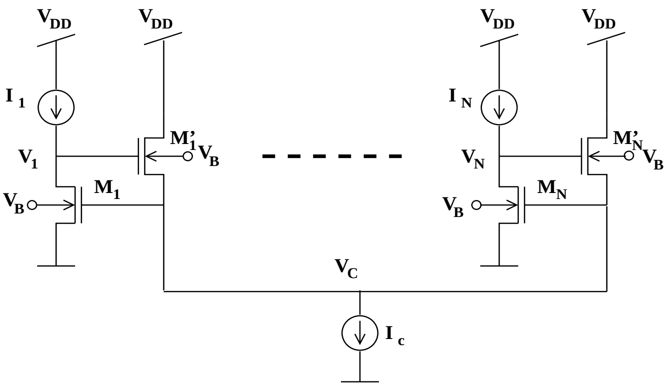

Fig. 1. Lazzaro WTA Circuit.

Our (W)inner-(T)ake-(A)ll network (see figure 1) consists of MOS transistors operating in subthreshold region. Due to its performances (low complexity, low power consumption, high speed) the circuit has been extensively used in many applications.

The circuit consists of N interacting cells. Each cell contains two nMOS transistors, M_k and M_k'. It receives a unidirectional input current I_k and produces an output voltage V_k. The circuit operates by choosing the maximum input current I_m and broadcasting its value as a voltage onto the global line V_c. Because all M transistors share the same gate to source voltage V_c, they should also sink the same current I_m. However, all I_j $(j \neq m)$ are smaller than I_m, as a result, the drain voltages V_j of all M_j decrease because of the Early effect. The decrease of V_j reduces the gate to source voltages of all M_j' transistors, hence decreasing the current through every M_j' transistor. As the summation of currents through all M' transistors is constant, equal to I_c, the current through M_m' increases. To accommodate this increment, the gate to source voltage of M_m' is forced to increase. As a result of the competition, the cell which receives the largest input current I_m has the highest output voltage V_m.

II. Preliminaries

Being fed with a list $I_1 > I_2 > \cdots > I_N$, the circuit might show a similarly ordered output list $V_1 > V_2 > \cdots > V_N$. Moreover, V_1 and V_2 should be split by known thresholds.

We show how to compute these thresholds and study their dependence on list density.

If we denote the gate by G, the source by S, the drain by D, and the bulk by B, we obtain the following static model for the transistor ([5]):

$$\left\{ \begin{array}{l} I_G = 0 \\ I_{DS} = \frac{W}{L} I_{D0} e^{\frac{V_{BS}(1-p)}{V_t}} \left(1 - e^{\frac{-V_{DS}}{V_t}}\right) e^{p\frac{V_{GS}-V_T}{V_t}} \end{array} \right. \quad (1)$$

Here $I_{D0} = \frac{K'2(pV_t)^2}{e^2}$.
The conditions for subthreshold operation are:

$$\left\{ \begin{array}{l} V_{GS} < V_T = V_{T0} + \gamma\left(\sqrt{\phi - V_{BS}} - \sqrt{\phi}\right) \\ 0 < V_{DS} \\ V_B < V_S \end{array} \right. \quad (2)$$

We denote $I_0 = \frac{W}{L} I_{D0} e^{V_B(1-p)/V_t}$ and we suppose I_0 is the same for all the transistors. Following, we write the currents for the k cell.
For the M_k transistor we have $V_S = 0$, $V_G = V_c$, $V_D = V_k$ and we obtain the equation:

$$I_{DSk} = I_0 e^{(pV_c - V_{T1})/V_t} \left(1 - e^{-V_k/V_t}\right) \quad (3)$$

with the conditions:

$$\left\{ \begin{array}{l} V_{DS} = V_k > 0 \\ V_{GS} = V_c < V_{T1} = V_{T0} + \gamma\left(\sqrt{\phi - V_B} - \sqrt{\phi}\right) \end{array} \right. \quad (4)$$

For the M_k' transistor we have $V_S' = V_c$, $V_G' = V_k$, $V_D' = V_{DD}$ and we obtain the equation:

$$I'_{DSk} = I_0 e^{(pV_k - V_{T2})/V_t}\left(e^{-V_c/V_t} - e^{-V_{DD}/V_t}\right) \quad (5)$$

with the conditions:

$$\begin{cases} V_{D'S'} = V_{DD} - V_c > 0 \\ V_{G'S'} = V_k - V_c < V_{T2} = V_{T0} + \gamma\left(\sqrt{\phi - V_B + V_c} - \sqrt{\phi}\right) \end{cases} \quad (6)$$

The stationary solution satisfies:

$$\begin{cases} I_j = I_{DSj} \\ \sum I'_{DSj} = I_c \end{cases} \quad (7)$$

These lead to the stationary circuit equations:

$$\begin{cases} e^{x_j} = \dfrac{1}{1 - i_j A(y)}\ , j \in \overline{1, N} \\ \dfrac{i_c}{g(y)} = \sum \dfrac{1}{(1 - i_j A(y))^p} \end{cases} \quad (8)$$

within the domain given by:

$$\begin{cases} y < y_M \\ 0 < x_j < f(y) = y + s(y) \\ y_0 < y < d \end{cases} \quad (9)$$

Here we have denoted: $x_j = V_j/V_t$, $y = V_c/V_t$, $i_j = I_j/I_0$, $i_c = I_c/I_0$, $a = V_{T1}/V_t = V_T(V_S = 0)/V_t$, $d = V_{DD}/V_t$, $s(y) = V_{T2}/V_t = V_T(V_S = V_c)/V_t$, $g(y) = e^{-ps(y)}(e^{-y} - e^{-d})$, $A(y) = e^{-py+ap}$, $y_M = min(a, d)$, $y_0 = f^{-1}(0)$.

III. Results

If $y_0 < y_m < y_M$, the next conditions guarantee that $(y, x_1, \cdots, x_N)$ belong to the domain in 9.

$$\begin{cases} I_M < [1 - e^{-f(y_m)}]e^{py_m - pa} \\ N\dfrac{g(y_M)}{1 - I_M A(y_M)} < i_c < N g(y_m) \end{cases} \quad (10)$$

Let us scale the currents to $[0, I_M]$. If the order of the currents is

$$0 \leq i_N < i_{N-1} < ... < i_2 < i_1 \leq I_M$$

we should have

$$0 \leq x_N < x_{N-1} < \cdots \leq Q_a \leq Q_b \leq x_1$$

Here Q_a and Q_b are two thresholds which necessarily should be list independent. To be more precise, let Δ be the smallest distance between currents, $i_j - i_{j+1} \geq \Delta$. We define $z = \Delta(N-1)/I_M$ a number between 0 and 1 called the "dispersion" of the list which shows how crowded the elements $i_1, \cdots, i_N$ are. A large z means a more dispersed list. Together with N (the number of list elements), z is an intrinsic parameter of data, i.e. its value does not depend on the confining interval $[0, I_M]$. We define:

$$\begin{aligned} Q_a &= min\{x_1 | \ (i_1, \cdots, i_N) \in L_{z0}\} \\ Q_b &= max\{x_2 | \ (i_1, \cdots, i_N) \in L_{z0}\} \end{aligned}$$

L_{z0} is the set of all accepted lists with $z \geq z_0$, i.e. z_0 is the minimum dispersion accepted by our WTA network. Our paper computes Q_a and Q_b and tries to show the conditions under which $Q_a \leq Q_b$, i.e. tries to find z_0.

Theorem 1 *If $\phi > V_t/4$, $y > V_B/V_t$ and $A(y^m)I_M \leq 1$ then the WTA selection works for $z > z_0$ where z_0 is the solution of:*

$$z e^{(1-z)\rho} - 1 + \frac{z}{N-1} = 0$$

$$\rho = \frac{2}{\dfrac{G'}{p^2} \cdot \dfrac{1}{A(y^M)I_M} + \displaystyle\sum_{k \neq 2} \dfrac{\frac{N-k}{N-1}}{\left(1 - A(y^M)I_M \frac{N-k}{N-1}\right)^{p+1}}}$$

$$G' = -\frac{g'(y^M)}{g(y^M)} \cdot \frac{N}{[1 - A(y_1)I_M]^p}$$

where $y_1 = y^M$ or y_M, the largest value of y.

We also have a result showing the values of list dispersion rendering the circuit unoperational:

Theorem 2 *If, $\phi > V_t/4$, $z < z_{00}$, where $z_{00} = \dfrac{1}{e^r + \frac{1}{N-1}}$ and $r = \dfrac{2p^2 A(y_m)I_M}{N[1 - A(y_m)I_M]^{p+1}}$ then the WTA property fails.*

IV. Conclusion

Conditions allowing the WTA operation are shown. They are explicitly expressed as a function of circuit and list parameters. An upper bound for the densities rendering the circuit non-operational is given as well.

References

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