Approximation of large-scale dynamical systems: An overview and some new results

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***Abstract*— In many applications one is faced with the task of simulating or controlling complex dynamical systems. Such applications include weather prediction, air quality management, VLSI chip design, molecular dynamics, micro-electro-mechanical systems (MEMS), etc. In all these cases complexity manifests itself as the number of first order differential or differential-algebraic equations which arise. For such systems, depending on the level of modeling detail required, complexity may range anywhere from a few thousand to a few million first order equations, and above. Simulating (controlling) systems of such complexity becomes a challenging problem, irrespective of the computational resources available. This talk aims at presenting an overview of issues arising in the simulation of complex dynamical systems together with some recent new results.**

Keywords—Large-scale systems, model reduction, SVD (Singular Value Decomposition) methods, Krylov methods, passivity preservation.

I. Introduction

Model reduction methods have evolved in several areas involving simulation and control of complex physical processes. The systems that inevitably arise in such simulations are often too complex to meet the expediency requirements of interactive design, optimization, or real time control. Model order reduction has been devised as a means to reduce the dimensionality of these complex systems to a level that is amenable to such requirements.

Model order reduction seeks to replace a large-scale system of differential or differential-algebraic equations by a system of substantially lower dimension that has nearly the same response characteristics. Generally, large systems arise due to accuracy requirements on the spatial discretization of the underlying PDEs (Partial Differential Equations). In the context of VLSI chip design, we have to deal with lumped approximations of distributed circuits arising in interconnects or packages.

Applications of model order reduction can be found in the

- Simulation of conservative systems, e.g., in molecular dynamics,
- Control and regulation of fluid flow (CFD: Computational Fluid Dynamics),
- Simulation and stabilization of large structures,
- VLSI chip design,
- Simulation of MEMS (Micro Electro Mechanical Systems),

to name but a few.

Various reduction techniques have been devised, but many of these are described in terms that are discipline- or application-specific even though they share common features and origins. This talk will bring together techniques from several fields and application areas in order to expose the similarities and common features of these approaches, and to address application-specific challenges.

II. Issues in Dimension Reduction

There are mainly two families of approximation methods namely, the *SVD-based* and *Krylov-based* approximation methods. The former family is based on the singular value decomposition and the second on moment matching. While the former has many desirable properties including an error bound, it cannot be applied to systems of high complexity. The strength of the latter on the other hand, is that it can be implemented iteratively and is thus appropriate for application to high complexity systems. An effort to combine the best attributes of these two families leads to a third class of approximation methods, which will be referred to as SVD/Krylov.

The problems in dimension reduction are challenging from the mathematical and algorithmic points of view. For example, the selection of appropriate basis functions in reduced-order basis approaches like proper orthogonal decomposition (POD) is highly problem-specific and requires a deeper mathematical understanding. On the algorithmic side there is a clear need for additional work in the area of large scale numerical linear algebra for instance in the solution of very large Lyapunov or Sylvester equations as well as in dealing with issues of sparsity. Moreover, it may be of interest to introduce non-traditional techniques such as wavelet bases.

Recently, a novel approach has been proposed which guarantees model reduction of DAE systems arising in circuit simulations, with preservation of passivity, irrespective of the structure of the equations involved. It belongs to the second family mentioned above, that is, it is a moment matching method, in which the so-called *spectral zeros* of the system play a key role. From a computational point of view it is crucial that this approach results in the solution of a structured large-scale eigenvalue problem which can be tackled efficiently.

III. Summary of Talk

In this talk problems in the range of topics listed above will be addressed. The hope is to stimulate discussion in

- the analysis and the development of the necessary mathematical theory,
- the extraction of the best features from different approaches with the goal of designing superior reduction methods, and
- the development of a deeper understanding of the application-specific challenges.

In the first part we will briefly describe some motivating examples, define the problem in mathematical terms and sketch several methodologies for its solution. The second part of the talk will present recent results concerning model reduction with preservation of passivity, an issue of great importance in interconnect simulations. We will conclude with open problems and directions for future research. For further details we refer to the publications listed below and the references therein.

References

[1] A.C. Antoulas, *Approximation of large-scale dynamical systems*, Advances in Design and Control, **DC06**, SIAM (Society for Industrial and Applied Mathematics), Philadelphia, 479 pages (2005).

[2] A.C. Antoulas, A new result on passivity preserving model reduction, *Systems and Control Letters*, **54**: 361-374 (2005).

[3] D.C. Sorensen, Passivity preserving model reduction via interpolation of spectral zeros, *Systems and Control Letters*, **54**: 347-360 (2005).