Electromagnetic Field Theory

5. Polarization, Magnetization, Conduction Power and mass transfer

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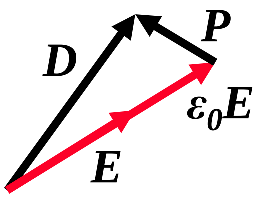
Polarization law

1. **Local form of the law:** $\mathbf{D} = \mathbf{f}(\mathbf{E})$



2. **Particular forms:**

- **in vacuum:** $\mathbf{D} = \varepsilon_0 \mathbf{E}, \quad \varepsilon_0 = \frac{1}{4\pi 9 \cdot 10^9} F/m$
- **in linear isotropic dielectrics:** $\mathbf{D} = \varepsilon \mathbf{E}, \quad \varepsilon_r = \varepsilon / \varepsilon_0 \Rightarrow \varepsilon = \varepsilon_r \varepsilon_0$
- **in non-isotropic dielectrics:** $\mathbf{D} = \overline{\overline{\varepsilon}} \mathbf{E}, \quad \overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}, \varepsilon_{ij} = \varepsilon_{ji}, \mathbf{ED} = \mathbf{E} \overline{\overline{\varepsilon}} \mathbf{E} > 0$
- **in permanent polarized bodies:** $\mathbf{D} = \overline{\overline{\varepsilon}} \mathbf{E} + \mathbf{P}_p$
- **in general:** $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \Rightarrow \mathbf{P} = \mathbf{D} - \varepsilon_0 \mathbf{E} = \mathbf{f}(\mathbf{E}) - \varepsilon_0 \mathbf{E} = \mathbf{P}_t(\mathbf{E}) + \mathbf{P}_p$
- **In linear dielectrics:** $\mathbf{P}_p = 0, \mathbf{P} = \mathbf{P}_t(\mathbf{E}) = \varepsilon_0 \chi \mathbf{E}, \mathbf{D} = \varepsilon_0 (1 + \chi) \mathbf{E} \Rightarrow \varepsilon_r = 1 + \chi$

3. **Physical meanings**:

- **Each substance has its own dielectric behavior**
- **Permanent polarization is a source of the electric field**
- **Due to their polarization, dielectrics perturbs the electric field**

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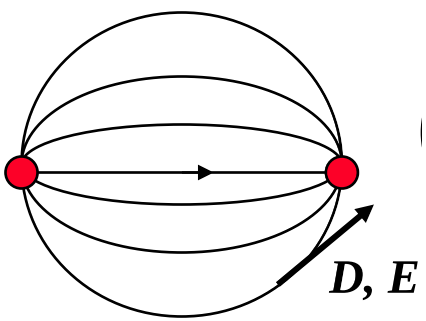
Electric field lines in substance

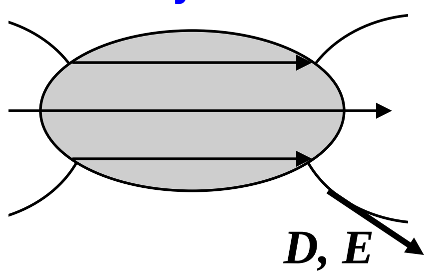
1. **In vacuum D and E have common lines (there are not necessary two vector field)**
2. **Permeable dielectrics concentrate and orientate the field lines**
3. **In an-isotropic dielectrics D and E have lines with different directions**
4. **In permanent polarized bodies:**

- D lines are continuous and closed having direction of P
- E lines are open with direction of D outside body and opposite to D and P inside body

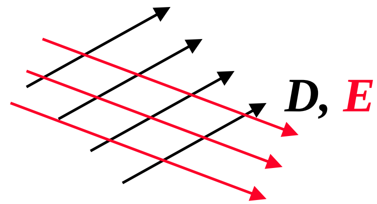


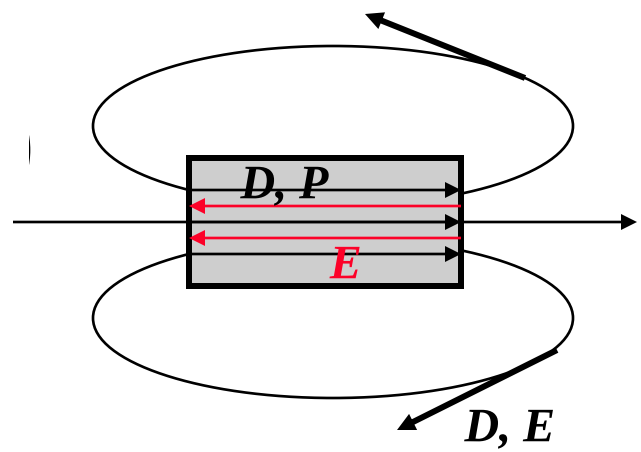
1. In vacuum

2. Dielectric body



3. In an-isotropic dielectrics





4. Permanent polarized body

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"Refraction" of the electric field lines

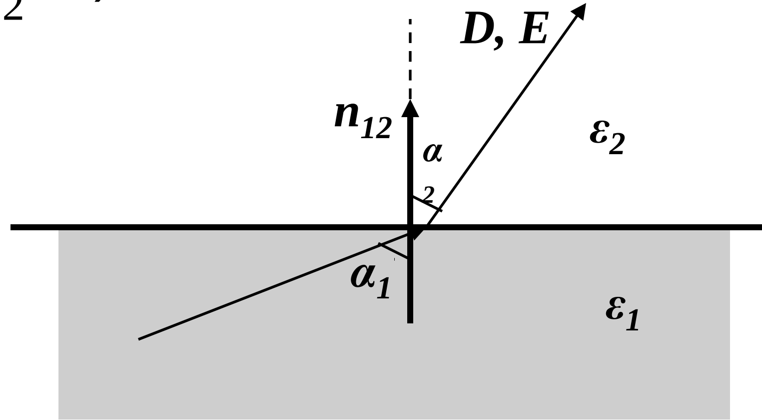
1. On a non-charged interface:

$$D_{n1} = D_{n2} \Rightarrow \varepsilon_1 E_{n1} = \varepsilon_2 E_{n2}$$

$$E_{t1} = E_{t2} \Rightarrow \varepsilon_1 E_{n1} / E_{t1} = \varepsilon_2 E_{n2} / E_{t2} \Rightarrow$$

$$\varepsilon_1 / tg\,\alpha_1 = \varepsilon_2 / tg\,\alpha_2 \Rightarrow$$

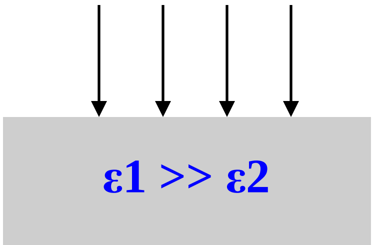
$$\frac{tg\,\alpha_1}{tg\,\alpha_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

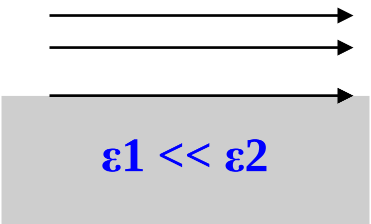


2. When ε1= ε1 the field is not perturbed (lines are not broken)

3. When ε1→ 0 (ε1 << ε2) α1→ 0 or α2→ π/2

4. When ε1→ infinity (ε1 >> ε2) α2→ 0 or α1→ π/2





- **Field avoids low permittivity bodies**
- **It s attracted by permeable bodies**

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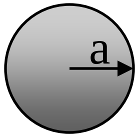
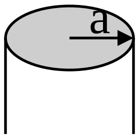
Applications of polarization law

1. Electric potential V and E,D fields of a dielectric sphere, uniformly charged placed in vacuum. Try a generalization.

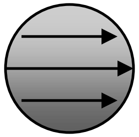


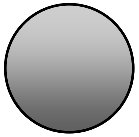
2. Electric potential V and E,D fields of a dielectric cylinder, uniformly charged placed in vacuum. Try a generalization.



3. Electric potential V and E,D fields of a dielectric plate, uniformly charged placed in vacuum. Try a generalization.

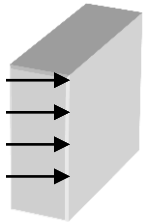
4. Electric potential V and E,D fields of a dielectric plate, uniformly and permanently polarized, placed in vacuum. Try a generalization.





5. Perturbation of uniform electric field due to an uncharged dielectric sphere. Try a generalization.

6. Electric potential V and E,D fields of a dielectric cylinder, uniformly and permanently polarized, placed in vacuum. Try a generalization.



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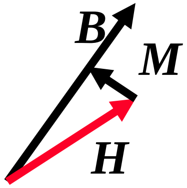
Magnetization law



1. Local form of the law:

$$\mathbf{B}=\mathbf{f}(\mathbf{H})$$

2. Particular forms:

- **in vacuum (non-magnetic):** $\mathbf{B}=\mu_0\mathbf{H},\quad \mu_0=4\pi\cdot 10^{-7}\,H/m$ – vacuum - permeabilitty
- **in linear isotropic media:** $\mathbf{B}=\mu\mathbf{H},\quad \mu_r=\mu/\mu_0 \Rightarrow \mu=\mu_r\mu_0$
- **in non-isotropic media:** $\mathbf{B}=\bar{\bar{\mu}}\mathbf{H},\quad \bar{\bar{\mu}}=\begin{bmatrix}\mu_{11} & \mu_{12} & \mu_{13}\\ \mu_{21} & \mu_{22} & \mu_{23}\\ \mu_{31} & \mu_{32} & \mu_{33}\end{bmatrix}, \mu_{ij}=\mu_{ji}, \mathbf{HB}=\mathbf{H}\bar{\bar{\mu}}\mathbf{H}>0$
- **in permanent mag. bodies:** $\mathbf{B}=\bar{\bar{\mu}}\mathbf{H}+\mathbf{I}_p, \mathbf{I}_p=\mu_0\mathbf{M}_p \Rightarrow \mathbf{B}=\bar{\bar{\mu}}\mathbf{H}+\mu_0\mathbf{M}_p=\mu_0(\bar{\bar{\mu}}_r\mathbf{H}+\mathbf{M}_p)$
- **in general:** $\mathbf{B}=\mu_0(\mathbf{H}+\mathbf{M}) \Rightarrow \mathbf{M}=\mathbf{B}/\mu_0-\mathbf{H}=\mathbf{M}_t(\mathbf{H})+\mathbf{M}_p$
- **In linear dielectrics:** $\mathbf{M}_p=0, \mathbf{M}=\mathbf{M}_t(\mathbf{H})=\chi\mathbf{H}, \mathbf{B}=\mu_0(1+\chi)\mathbf{H} \Rightarrow \mu_r=1+\chi$

3. Physical meanings:

- **Each substance has its own magnetic behavior**
- **Permanent magnetization is a source of magnetic field**
- **Due to magnetization, the magnetic field is perturbed**

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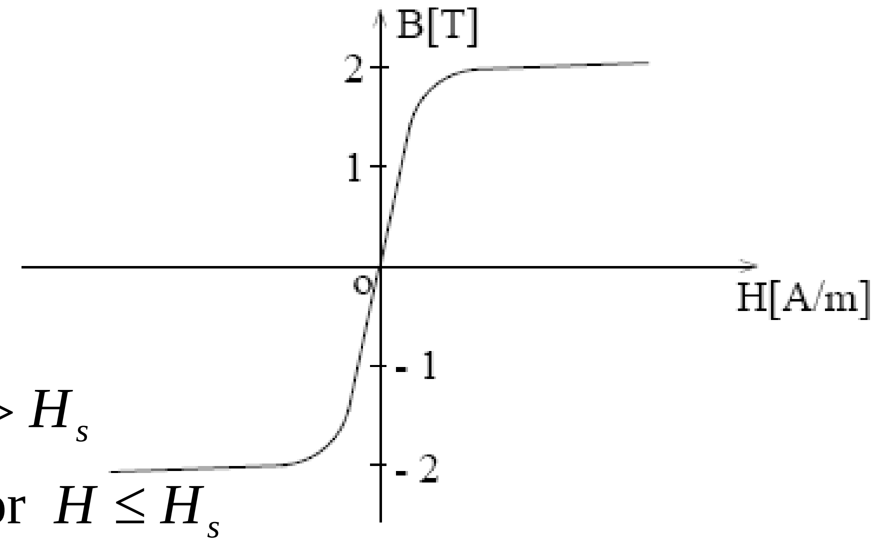
Ferromagnetic materials

1. **Soft magnetic materials:**

B || H, B = f(H) f:$R_+ \rightarrow$ R magnetization characteristic



$$B = f(H) = \begin{cases} \mu_r \mu_0 H, \text{for } H \le H_s \\ \mu_0 (H + (\mu_r - 1) H_s), \text{for } H > H_s \end{cases}$$

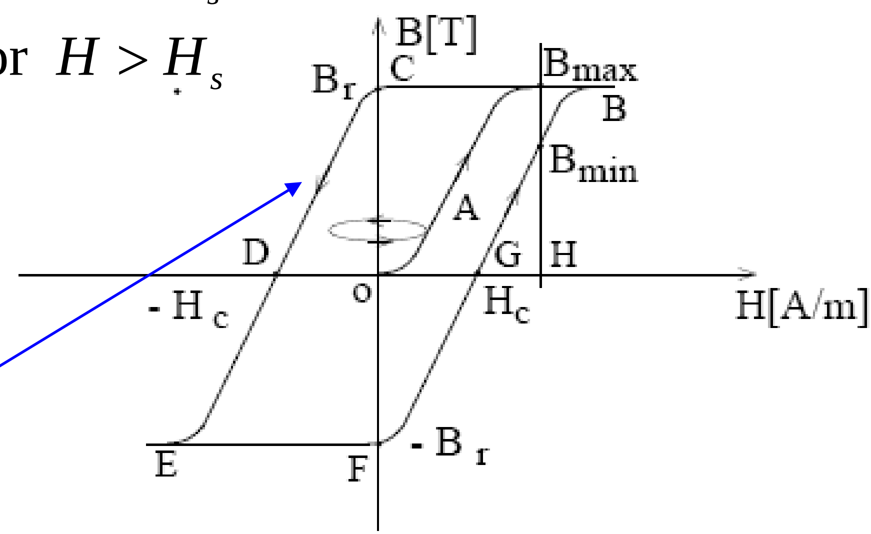
$$- \quad M = f(H)/\mu_0 - H = \begin{cases} \chi H = (\mu_r - 1) H, \text{for } H \le H_s \\ H_s = (\mu_r - 1) H_s, \text{for } H > H_s \end{cases}$$

$$\mu_r = 100 - 100000, B_s = \mu_r \mu_0 H_s = 0.5 \ldots 2T$$

2. Hard magnetic materials.

Magnetic hysterezis.

In permanent magnets: B =μ H+ μ_0 M_p



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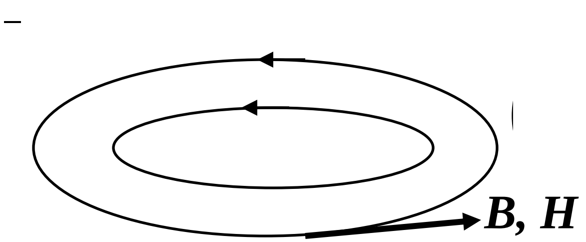
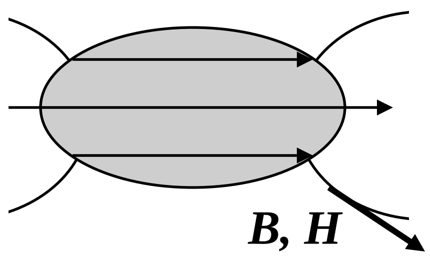
Magnetic field lines in substance

1. In vacuum B and H have common lines **(there are not necessary two vector field)**
2. Ferromagnetic bodies concentrate and orientate the magnetic field lines
3. B and H may have lines with different directions, orientations or densities
4. In permanent magnets:

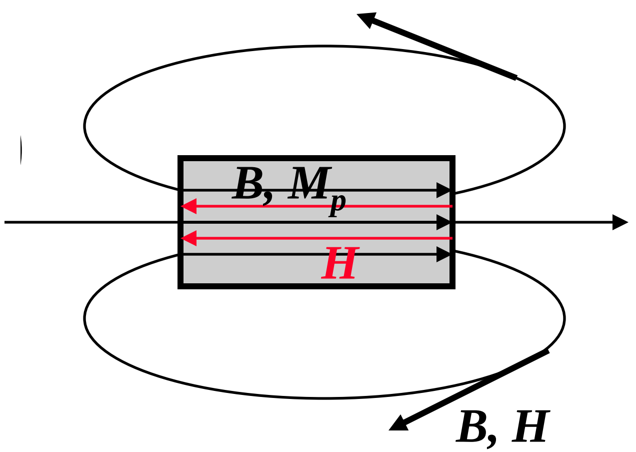
- B lines are continuous and closed having direction of M_p
- H lines are open with direction of B outside body and opposite to B and M_p inside body (demagnetized field)



In vacuum



In soft materials



Field of a permanent magnet

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"Refraction" of the magnetic field lines

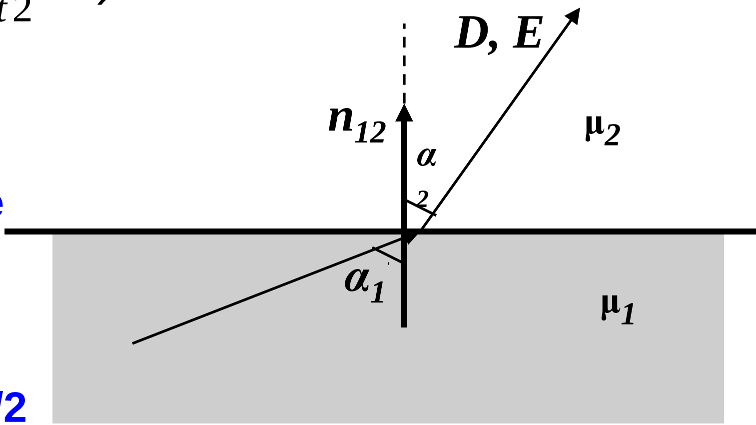
1. On a interface between two media:

$$\frac{tg\,\alpha_1}{tg\,\alpha_2} = \frac{\mu_1}{\mu_2}$$

$$B_{n1} = B_{n2} \Rightarrow \mu_1 H_{n1} = \mu_2 H_{n2}$$

$$H_{t1} = H_{t2} \Rightarrow \mu_1 H_{n1} / H_{t1} = \mu_2 H_{n2} / H_{t2} \Rightarrow$$

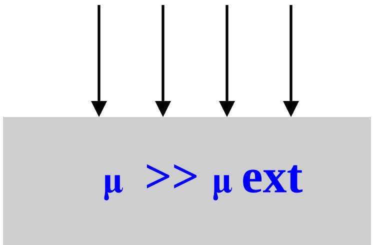
$$\mu_1 / tg\,\alpha_1 = \mu_2 / tg\,\alpha_2 \Rightarrow$$



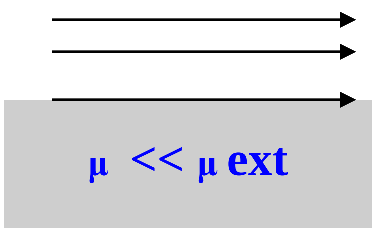
2. When μ1= μ 1 the field is not perturbed (lines are not broken)

3. When μ 1→ 0 (μ 1 << μ 2) α1→ 0 or α2→ π/2

4. When μ 1→ infinity (μ 1 >> μ 2) α2→ 0 or α1→ π/2



Perfect ferromagnetic body
(1/ μ →0, Hint = 0)



A-magnetic body
(μ =0, Bint = 0)

- Field avoids low permeable bodies
- It s attracted by permeable bodies

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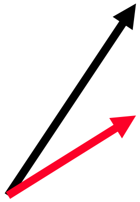
Conduction. Ohm's law

1. **Local form of the law:** $$\mathbf{J} = \mathbf{f}(\mathbf{E})$$

2. **Particular forms:**

- **in vacuum** $\mathbf{J} = 0$
- **in linear isotropic conductors** $\mathbf{J} = \sigma\mathbf{E} \Leftrightarrow \mathbf{E} = \rho\mathbf{J}, \text{with } \rho = 1/\sigma$
- **in non-isotropic conductors** $\mathbf{J} = \overline{\overline{\sigma}}\mathbf{E}$
- **in bodies with intrinsic field** $\mathbf{J} = \overline{\overline{\sigma}}(\mathbf{E} + \mathbf{E}_i)$



3. Physical meanings:

- **The current is generated by the electric field**
- **Each substance has its own behavior from conduction point of view**
- **Electric field may have sources of non-EM nature**

4. The lines of the generated electric field

- **May be open**
- **They have the tendency to be opposite to the intrinsic field Ei**

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Several forms of material (constitutive) laws

Field:	Polarization	Magnetization	Conduction
General	$\mathbf{D} = \mathbf{f}(\mathbf{E})$	$\mathbf{B} = \mathbf{f}(\mathbf{H})$	$\mathbf{J} = \mathbf{f}(\mathbf{E})$
Vacuum	$\mathbf{D} = \varepsilon_0 \mathbf{E},$	$\mathbf{B} = \mu_0 \mathbf{H}$	$\mathbf{J} = 0$
Linear izotropic	$\mathbf{D} = \varepsilon \mathbf{E},$	$\mathbf{B} = \mu \mathbf{H},$	$\mathbf{J} = \sigma \mathbf{E}$
Linear anizotropic	$\mathbf{D} = \overline{\overline{\varepsilon}} \mathbf{E},$	$\mathbf{B} = \overline{\overline{\mu}} \mathbf{H},$	$\mathbf{J} = \overline{\overline{\sigma}} \mathbf{E}$
Affine	$\mathbf{D} = \overline{\overline{\varepsilon}} \mathbf{E} + \mathbf{P}_p$	$\mathbf{B} = \overline{\overline{\mu}} \mathbf{H} + \mu_0 \mathbf{M}_p$	$\mathbf{J} = \overline{\overline{\sigma}} (\mathbf{E} + \mathbf{E}_i)$
Nonlinear	$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} =$ $\varepsilon_0 \mathbf{E} + \mathbf{P}_t(\mathbf{E}) + \mathbf{P}_p$	$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) =$ $\mu_0 (\mathbf{H} + \mathbf{M}_t(\mathbf{H}) + \mathbf{M}_p)$	$\mathbf{J} = \overline{\overline{\sigma}} (\mathbf{E} + \mathbf{E}_i(\mathbf{E}))$
Field line refraction	$\dfrac{tg\,\alpha_1}{tg\,\alpha_2} = \dfrac{\varepsilon_1}{\varepsilon_2}$	$\dfrac{tg\,\alpha_1}{tg\,\alpha_2} = \dfrac{\mu_1}{\mu_2}$	$\dfrac{tg\,\alpha_1}{tg\,\alpha_2} = \dfrac{\sigma_1}{\sigma_2}$

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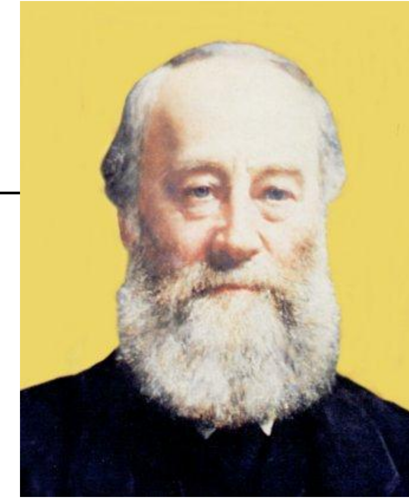
Joule's law of power transfer

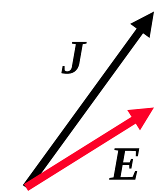


1. **Local form of the law:** $p = \mathbf{E} \cdot \mathbf{J}$ [W/m^3]

2. **Particular forms:**

- **in linear conductors:** $p = \mathbf{E} \cdot \mathbf{J} = \sigma E^2 = \rho J^2 \geq 0$
- **in nonlinear conductors:**

$$p = \mathbf{E} \cdot \mathbf{J} = \sigma(\mathbf{E} + \mathbf{E}_i)\mathbf{E} = \sigma \mathbf{E}^2 + \sigma \mathbf{E}\mathbf{E}_i =$$
$$(\rho \mathbf{J} - \mathbf{E}_i)\mathbf{J} = \rho \mathbf{J}^2 - \mathbf{E}_i \mathbf{J}$$



3. **Physical meanings:**
- **The conduction implies a power transfer between field and substance**
- **The electric current generates heat**
- **Transfer of power from field to substance in linear conductors is a non-reversible process**

4. **Definition of the volumetric power density:**

$$p = \lim_{\Delta V \to 0} \frac{\Delta P}{\Delta V} \; cu \; \Delta P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t},$$

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Faraday's law of mass transfer

1. **Local form of the law:**

$$\boldsymbol{\delta} = k\mathbf{J} \quad [\mathrm{kg/m^2 s}], \quad k = \begin{cases} 0, \text{ in metals} \\ A/F_0 z \text{ in electrolyts} \end{cases}$$

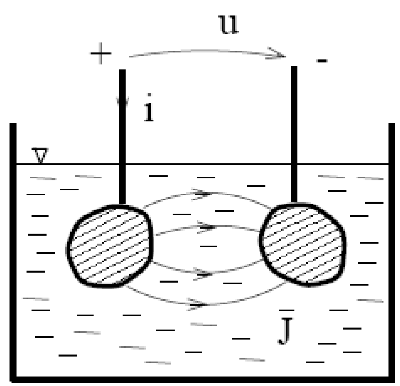
2. **The global form – electrolysis law**

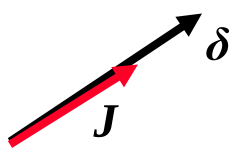
$$F_0 = 96490C - \text{Faraday's constant}$$

$$Q_m = \int_S \boldsymbol{\delta} dA = \int_S k\mathbf{J}\, dA$$

$$m = \int_{t1}^{t2} Q_m dt = \int_{t1}^{t2}\int_S k\mathbf{J}\, dA dt =$$

$$k\int_{t1}^{t2}\int_S \mathbf{J}\, dA dt = k\int_{t1}^{t2} i dt = kIt$$





3. Physical meanings:

- The conduction implies a mass transfer along the electric current
- The flux density of the mass transfer is proportional to J
- The proportionality coefficient is a material constant

4. Definition of the mass transfer flux density:

$$\delta = \mathrm{n} \cdot \lim_{\Delta A \to 0, \Delta t = 0} \frac{\Delta m}{\Delta A \cdot \Delta t}$$

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Not so easy questions for curious people

1. When $\psi = C\,u$ (the global form of the polarization law) is valid ?
2. When $\varphi = \Lambda\, U_m$ (the global form of the magnetization law) is valid ?
3. When $i = G\,u$ (the global form of the Ohm's law) is valid ?
4. When $p = ui$ – (the global form of the Joule's law) is valid ?
5. How looks like the expressions of material laws in the case of linear/nonlinear, isotropic/anisotropic, homogeneous/non-homogeneous media ? (try all combinations)
6. How can be obtained the best affine approximation of the nonlinear constitutive relations ?
7. How can be expressed the B-H relation in the case of nonlinear, anisotropic soft ferromagnetic materials ?
8. How can be expressed in mathematical terms the B-H relation in the case of hysteretic ferromagnetic materials ?
9. May depend the current density w.r.t. magnetic field J(E, B) ?
10. Find the constitutive relations for all materials in the room you are now.

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