Electromagnetic Modeling

13. ElectroDynamic Fields and Wives

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Maxwell's equations

1. They are the differential form of the EM field laws in non-moving media:

$$\nabla \cdot \mathbf{D} = \rho$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

2. In addition, to obtain a complete system of equations, the constitutive relations should be added

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}(\mathbf{E}) = \varepsilon \mathbf{E} \text{ in linear dielectrics}$$
$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}(\mathbf{H})) = \mu \mathbf{H} \text{ in linear media}$$
$$\mathbf{J} = \sigma (\mathbf{E} + \mathbf{E}_i(\mathbf{E})) = \sigma \mathbf{E} \text{ in linear conductors}$$

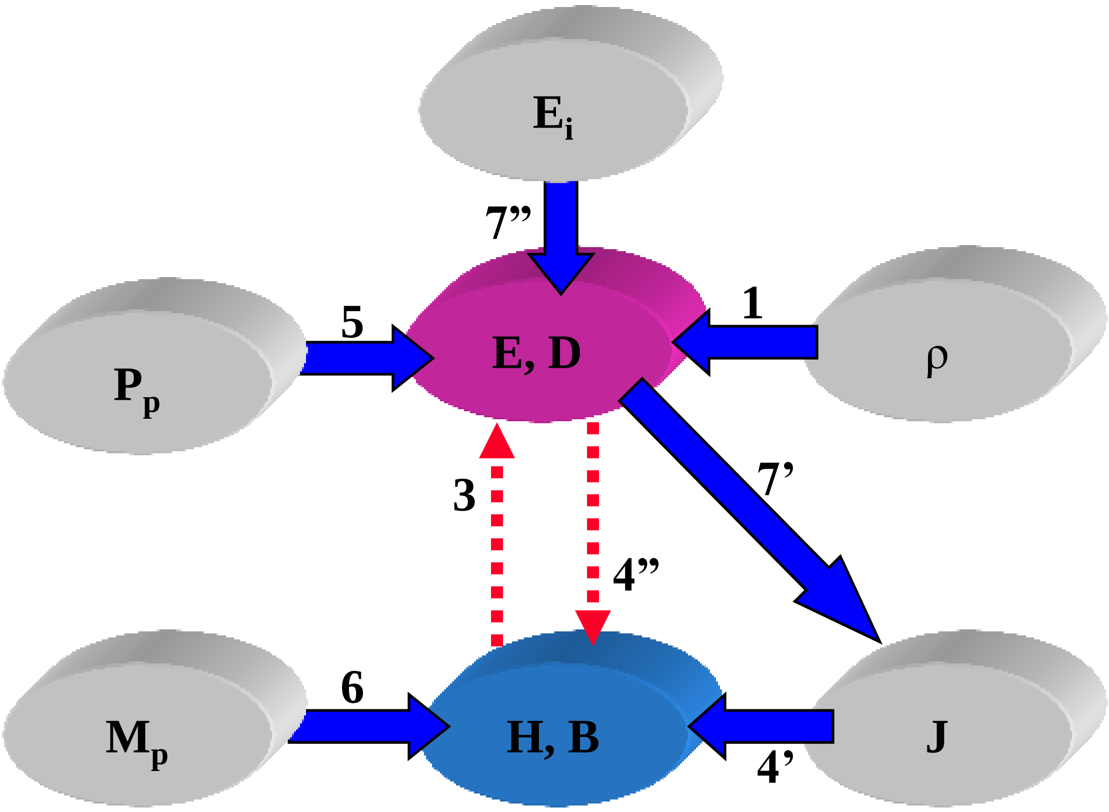
3. In vacuum, the EM field is described only by E and H:

$$\nabla \cdot \mathbf{D} = 0$$
$$\nabla \cdot \mathbf{H} = 0$$
$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$
$$\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

They are compatible with the Einstein's theory of relativity, being invariant in inertial moving reference systems (invariant to the Lorentz transform)

Diagram of fundamental EM phenomena (causal relations)

1. $\nabla \cdot \mathbf{D} = \rho$
2. $\nabla \cdot \mathbf{B} = 0$
3. $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$
4. $\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$
5. $\mathbf{D} = \varepsilon\, \mathbf{E} + \mathbf{P}_p(\mathbf{E})$
6. $\mathbf{B} = \mu\, (\mathbf{H} + \mathbf{M}_p(\mathbf{H}))$
7. $\mathbf{J} = \sigma(\mathbf{E} + \mathbf{E}_i(\mathbf{E}))$



Full Wave Loss-Less regime

$$\nabla \cdot \mathbf{D} = \rho$$

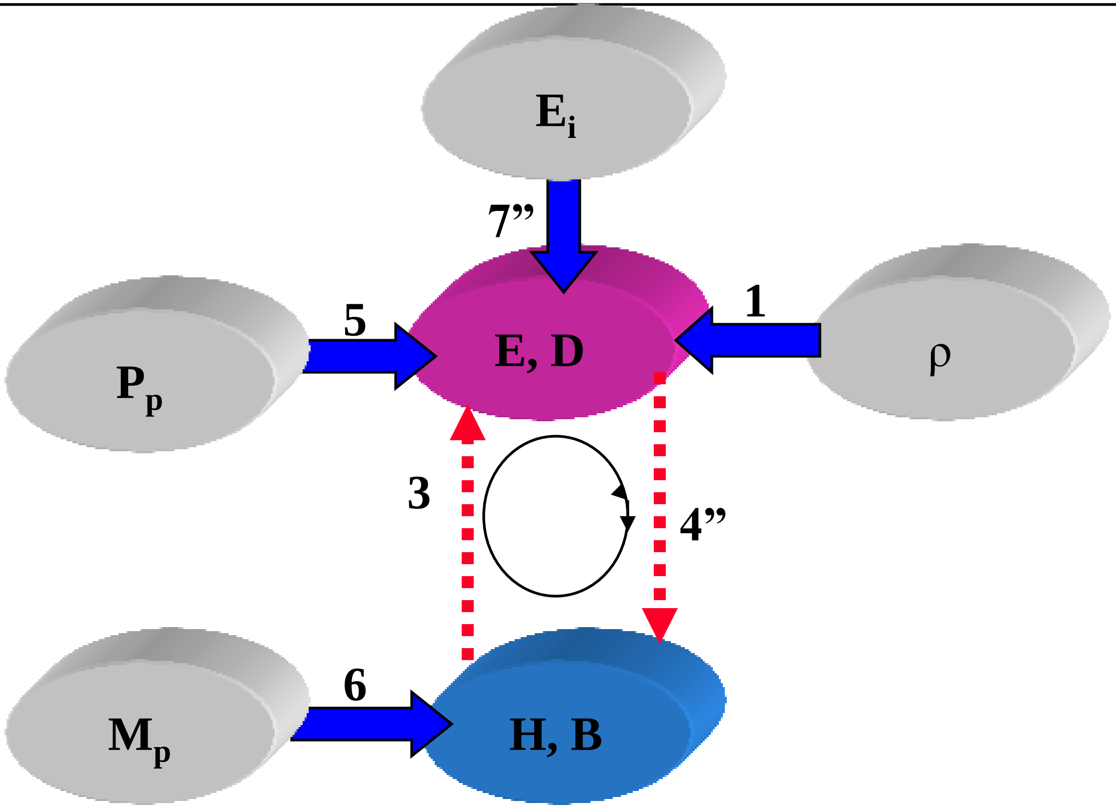
$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$$

$$\mathbf{D} = \varepsilon\, \mathbf{E} + \mathbf{P}_p(\mathbf{E})$$

$$\mathbf{B} = \mu\, (\mathbf{H} + \mathbf{M}_p(\mathbf{H}))$$



Sources: $E_{i,}$, M_p, P_p,
Fields: E, D, H, (div-free) B
Potentials: A, *V*
Material constants: ε, μ
PDE of hyperbolic type
Field is propagating in space

LL hypothesis:

1. **No movement**
2. **No conductive losses (σ=0, J=0)**
3. **No hysteretic losses of dielectric or magnetic nature**

Electro-dynamic potentials, second order equations

$$
\begin{aligned}
&\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \;\Rightarrow\; \nabla \times (\nabla \times \mathbf{E}) = -\mu\sigma \frac{\partial \mathbf{E}}{\partial t};\\
&\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \;\Rightarrow\; \nabla \times (\nabla \times \mathbf{H}) = \sigma \nabla \times \mathbf{E} + \varepsilon \frac{\partial \nabla \times \mathbf{E}}{\partial t} \Rightarrow \\
&\Rightarrow \nabla \times (\nabla \times \mathbf{H}) = -\mu\sigma \frac{\partial \mathbf{H}}{\partial t} - \varepsilon\mu \frac{\partial^2 \mathbf{H}}{\partial t^2}.
\end{aligned}
\tag{1.150}
$$

Potenţialele vector şi scalar satisfac în acest regim ecuaţiile:

Ecuatii de tip hiperbolic

$$
\begin{aligned}
\Delta \mathbf{A} - \varepsilon\mu \frac{\partial^2 \mathbf{A}}{\partial t^2} &= -\mu \mathbf{J};\\
\Delta V - \varepsilon\mu \frac{\partial^2 V}{\partial t^2} &= -\frac{\rho}{\varepsilon},
\end{aligned}
\tag{1.151}
$$

în care intervin mărimile cu distribuţie necunoscuta $\mathbf{J}$ si ρ. Dacă acestea se exprimă în funţie de câmpul electric se obţin ecuaţiile:

$$
\begin{aligned}
\Delta \mathbf{A} - \varepsilon\mu \frac{\partial^2 \mathbf{A}}{\partial t^2} &= -\sigma \left(\nabla V + \frac{\partial \mathbf{A}}{\partial t} \right);\\
\nabla A &= \varepsilon\mu \frac{\partial V}{\partial t},,
\end{aligned}
\tag{1.152}
$$

Fundamental problem, solution uniqueness

Maxwell equations in linear media with μ, ε, σ>0 and permanent sources:

$$\begin{cases} \mathbf{rot}\ \boldsymbol{E} = -\dfrac{\partial \boldsymbol{B}}{\partial t}, & \text{div}\ \boldsymbol{D} = \rho_V, & \boldsymbol{D} = \varepsilon \boldsymbol{E} + \boldsymbol{P}_p & \mathrm{J} = \sigma(\mathrm{E} + \mathrm{E}_i) \\ \mathbf{rot}\ \boldsymbol{H} = \boldsymbol{J} + \dfrac{\partial \boldsymbol{D}}{\partial t}, & \text{div}\ \boldsymbol{B} = 0, & \boldsymbol{B} = \mu \boldsymbol{H} + \mu_0 \boldsymbol{M}_p \end{cases}$$

have a unique solution in D bounded by Σ for any $0<t<T$, if there are known:

- Field internal sources (CS): $\boldsymbol{Pp}(\mathbf{r},t)$; $\boldsymbol{Mp}(\mathbf{r},t)$, $\mathbf{r} \in D$, $t \in [0,T]$;
  $\boldsymbol{Ji}(\boldsymbol{r},t) = \sigma \boldsymbol{Ei}(\boldsymbol{r},t)$, $\boldsymbol{r} \in D$, $t \in [0,T]$;
- Boundary conditions on Σ (CΣ): $\boldsymbol{E_t}(\boldsymbol{r},\boldsymbol{t})$, $\boldsymbol{r} \in S_E$; $\boldsymbol{H_t}(\boldsymbol{r},\boldsymbol{t})$, $\boldsymbol{r} \in S_H = \Sigma - S_E$, $t \in [0,T]$;
- Initial conditions (C0): $\boldsymbol{D}(\boldsymbol{r},0)$; $\boldsymbol{B}(\boldsymbol{r},0)$, $\boldsymbol{r} \in D$, for $t = 0$.

The prove is based on the lemma of trivial solution:

Maxwell equations with zero CS, CΣ, C0 have only zero solution

$$\int_D \sigma \mathrm{E}\cdot\mathrm{E}\, dV + \frac{\partial}{\partial t}\int_{D_\Sigma}\left(\frac{\mu \mathrm{H}\cdot\mathrm{H}}{2} + \frac{\varepsilon \mathrm{E}\cdot\mathrm{E}}{2}\right)dV = \int_D \mathrm{E}\cdot\mathrm{J}_i\, dV - \int_{D_\Sigma}\left(\mu_0 \mathrm{H}\cdot\frac{\partial \mathrm{M}_p}{\partial t} + \mathrm{E}\cdot\frac{\partial \mathrm{P}_p}{\partial t}\right)dV - \oint_\Sigma (\mathrm{E}\times\mathrm{H})\cdot\mathbf{n}\, dS$$

$$\int_D \sigma \mathrm{E}^2 dV + \frac{1}{2}\frac{\partial}{\partial t}\int_{D_\Sigma}\left(\mu \mathrm{H}^2 + \varepsilon \mathrm{E}^2\right)dV = 0 \Rightarrow 0 \le \int_{D_\Sigma}\left(\mu \mathrm{H}^2 + \varepsilon \mathrm{E}^2\right)dV = -2\int_0^t\int_D \sigma \mathrm{E}^2 dV \le 0 \Rightarrow \mathrm{H}=0, \mathrm{E}=0 \Rightarrow$$

$$\mathrm{D}=0, \mathrm{B}=0, \mathrm{J}=0, \rho=0$$

Loss-less full wave EM field

În **regim general variabil fără pierderi (EDFP)**, câmpul electromagnetic este descris de ecuaţii cu derivate parţiale de tip hiperbolic, de forma:

$$
\begin{aligned}
&\nabla \times \left(\frac{1}{\mu}\nabla \times \mathbf{E}\right) + \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \ \text{ in } \ \Omega, \\
&\hat{\mathbf{n}} \times \mathbf{E} = 0, \ \text{ pe } \ S_E, ; \\
&\hat{\mathbf{n}} \times \mathbf{H} = 0, \ \text{ pe } \ S_H, ; \\
&\mathbf{E}\left(\mathbf{r}, t = 0\right) = \mathbf{E}_0\left(\mathbf{r}\right), \ \text{ in } \ \Omega, \\
&\left.\frac{\partial \mathbf{E}\left(\mathbf{r}, t\right)}{\partial t}\right|_{t=0} = \mathbf{E}_0'\left(\mathbf{r}\right), \ \text{ in } \ \Omega.
\end{aligned}
\tag{3.165}
$$

la care sunt adăugate condiţii de frontieră (pentru $\mathbf{E}_t$ sau $\mathbf{H}_t$), dar şi două condiţii iniţiale, una pentru intensitatea câmpului şi alta pentru derivata ei faţă de timp, la momentul $t = 0$.

Open boundary conditions

- **ABC – Absorbing Boundary Condition, e.g. Mur b.c.**

$$(\partial_x - c_0^{-1}\partial_t)W|_{x=0} = 0.$$

Were W are field components, or second order approximation - Enquist Majda b.c.

$$(c_0^{-1}\partial_{xt}^2 - c_0^{-2}\partial_t^2 + \tfrac{1}{2}(\partial_y^2 + \partial_z^2))W|_{x=0} = 0.$$

–

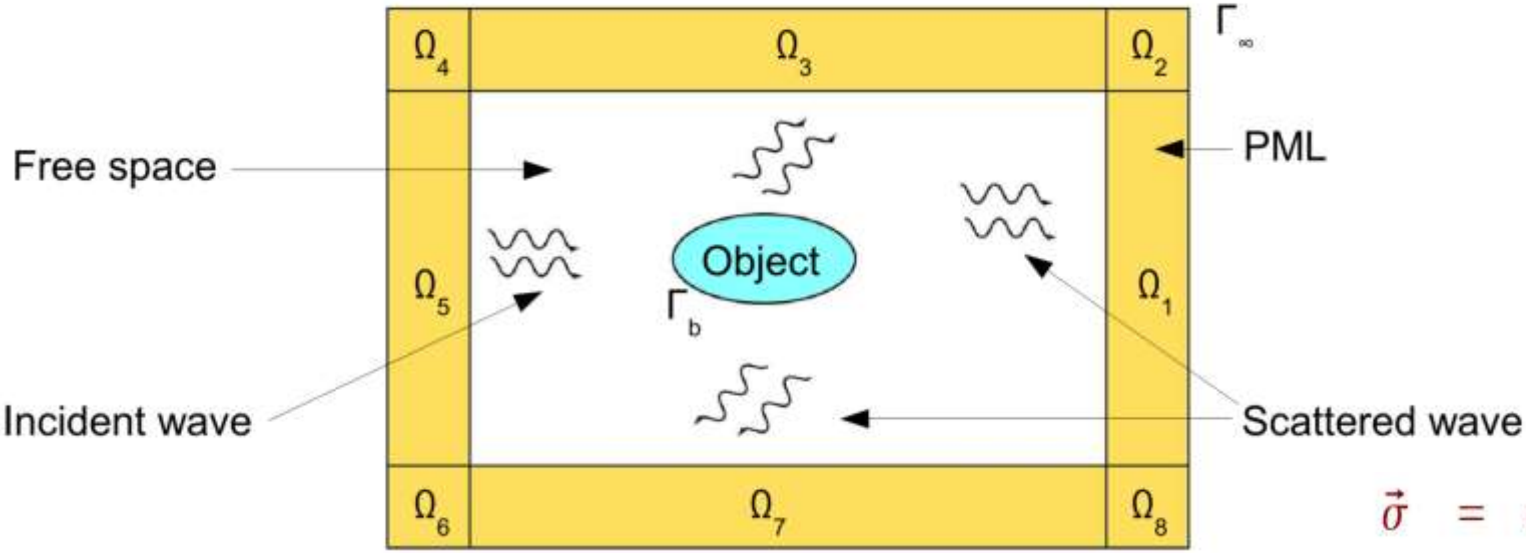
$$(\partial_x E_z - c_0^{-1}\partial_t E_z - (c_0\mu_0/2)\partial_y H_x)_{x=0} = 0$$

the fields do not depend on z and are E-polarized, i.e., $E = E_z i_z$ and $H = H_x i_x + H_y i_y$, (12) applies to E_z only. Now, in

- **PML – Perfect Matched Layer, e.g. Berenger's b.c.**

- **FEM-BEM – Coupled Finite Element – Boundary Element Method, e.g. EFIE – Electric Field Integral Equations or MFIE – Magnetic Field Integral Equations (see harmonic fields)**

PML – Perfect Matched Layer



$$\vec{\sigma} = \sigma_x \vec{e}_x + \sigma_y \vec{e}_y$$

$$\vec{\sigma}_1 = \sigma_0 \left| \frac{x-a}{A-a} \right|^n \vec{e}_x$$

$$\vec{\sigma}_3 = \sigma_0 \left| \frac{y-b}{B-b} \right|^n \vec{e}_y$$

$$\vec{\sigma} = \vec{\sigma}_1 \text{ in } \Omega_1$$

$$\vec{\sigma} = \vec{\sigma}_3 \text{ in } \Omega_3$$

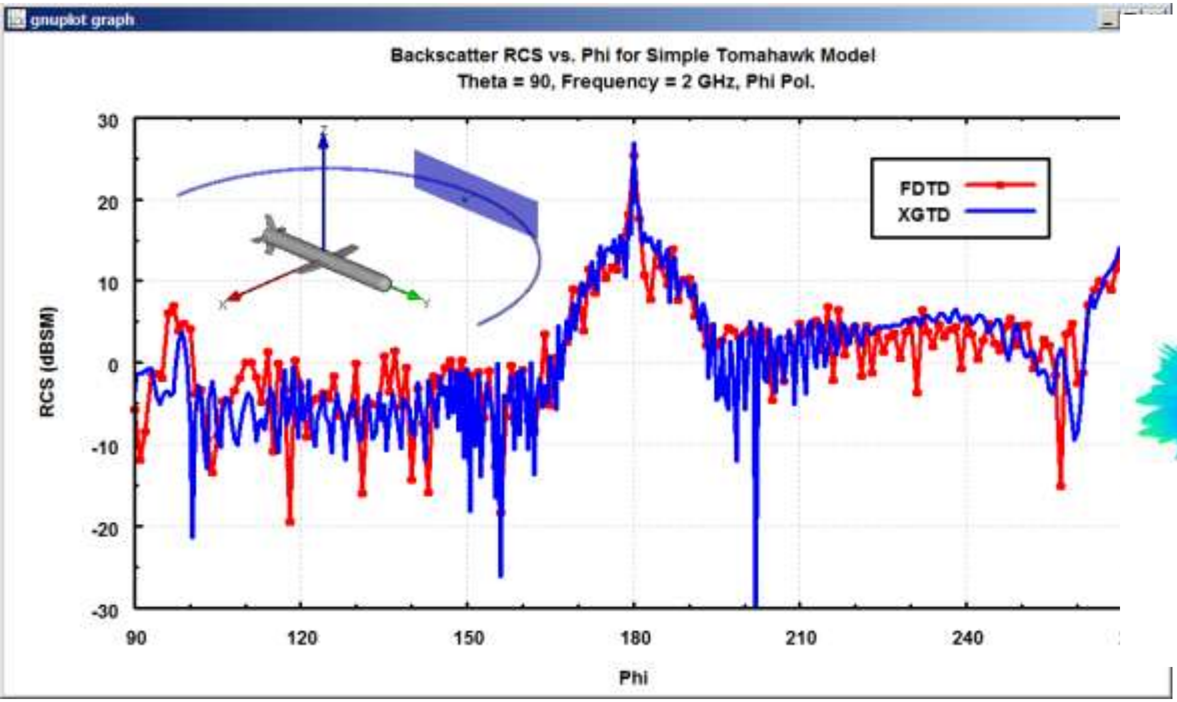
$$\vec{\sigma} = \vec{\sigma}_1 + \vec{\sigma}_3 \text{ in } \Omega_2$$

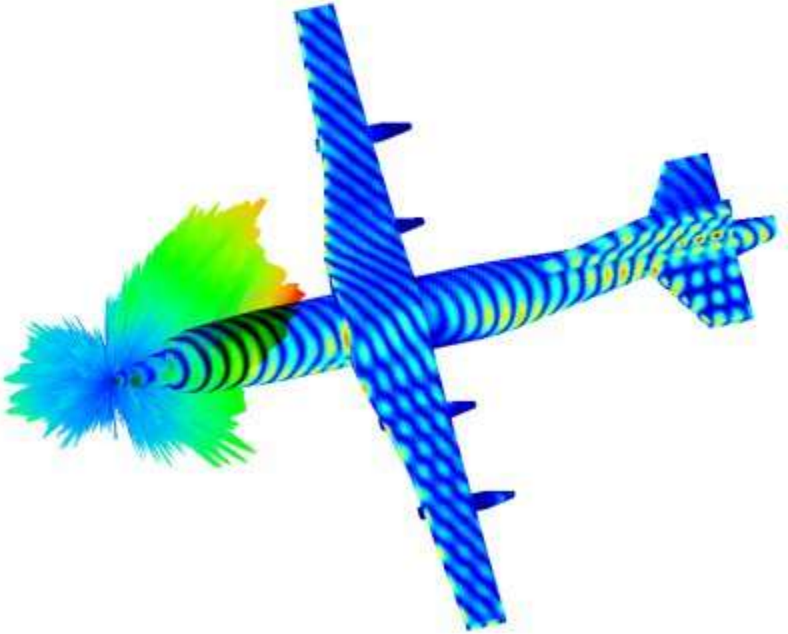
In PML: electric and “magnetic” (artificial) conductivity, or an anisotropic absorber in PML with material tensor:

$$\mu \frac{\partial \vec{H}}{\partial t} + \nabla X \vec{E} + \sigma_H \vec{H} = 0$$

$$\varepsilon \frac{\partial \vec{E}}{\partial t} - \nabla X \vec{H} + \sigma_E \vec{E} = 0$$

EM waves scattering. Radar Cross Section





Consider a plane wave $\boldsymbol{E}^i$ incident on a perfectly conducting object. The incident wave produces surface currents $\boldsymbol{J}_s$ on the conductor, which generate a scattered electric field $\boldsymbol{E}^s$. The scattered field is determined by the boundary condition

$$\hat{\boldsymbol{n}} \times (\boldsymbol{E}^i + \boldsymbol{E}^s) = 0, \qquad \boldsymbol{r} \in \partial\Omega_c, \tag{7.27}$$

which states that the total tangential electric field vanishes on the conductor surface $\partial\Omega_c$. This is used for the electric field integral equation.

Weak formulation

Maxwell's equations can be put in variational form in a few different ways. One way is to apply the general prescription (6.92) to the lossless self-adjoint curl-curl equation

$$\nabla \times \mu^{-1} \nabla \times \boldsymbol{E} + \epsilon \partial^2 \boldsymbol{E} / \partial t^2 = -\partial \boldsymbol{J} / \partial t, \tag{6.104}$$

integrate both in space and time, and ignore the boundary terms. This gives the quadratic form

$$L = \iint \left(\frac{1}{2\mu} |\nabla \times \boldsymbol{E}|^2 - \frac{\epsilon}{2} \left| \frac{\partial \boldsymbol{E}}{\partial t} \right|^2 + \boldsymbol{E} \cdot \frac{\partial \boldsymbol{J}}{\partial t} \right) dV\, dt. \tag{6.105}$$

For a small variation of the electric field $\boldsymbol{E} \to \boldsymbol{E} + \delta \boldsymbol{E}$, the first-order change of L is

$$\delta L = \iint \left(\frac{1}{\mu} \nabla \times \boldsymbol{E} \cdot \nabla \times \delta \boldsymbol{E} - \epsilon \frac{\partial \boldsymbol{E}}{\partial t} \cdot \frac{\partial \delta \boldsymbol{E}}{\partial t} + \frac{\partial \boldsymbol{J}}{\partial t} \cdot \delta \boldsymbol{E} \right) dV\, dt,$$

and an integration by parts (ignoring boundary terms) gives

$$\delta L = \iint \left(\nabla \times \frac{1}{\mu} \nabla \times \boldsymbol{E} + \epsilon \frac{\partial^2 \boldsymbol{E}}{\partial t^2} + \frac{\partial \boldsymbol{J}}{\partial t} \right) \cdot \delta \boldsymbol{E}\, dV\, dt.$$

Thus, if $\boldsymbol{E}$ is a solution of Maxwell's equations, then $\delta L = 0$ for any $\delta \boldsymbol{E}$, which means that $L[\boldsymbol{E}]$ is stationary. Conversely, to make L stationary, i.e., $\delta L = 0$ for an arbitrary $\delta \boldsymbol{E}$, the curl-curl equation (6.104) must be satisfied.

Weak formulation

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}, \quad \boldsymbol{E} = -\nabla\phi - \frac{\partial \boldsymbol{A}}{\partial t}. \tag{6.106}$$

The quadratic form is the magnetic minus the electric energy, plus terms involving the sources, integrated in space and time:

$$L = \iint \left(\frac{B^2}{2\mu} - \boldsymbol{A}\cdot\boldsymbol{J} - \frac{\epsilon E^2}{2} + \rho\phi \right) dV\,dt. \tag{6.107}$$

We get Maxwell's equations by setting the first variation of L with respect to ϕ and $\boldsymbol{A}$ to zero. For $\phi \to \phi + \delta\phi$, integration by parts gives the first variation

$$\begin{aligned} \delta L &= \iint (\epsilon \nabla\delta\phi \cdot \boldsymbol{E} + \rho\delta\phi)\; dV\,dt \\ &= \iint (\rho - \nabla\cdot\epsilon\boldsymbol{E})\,\delta\phi\, dV\,dt = 0. \end{aligned} \tag{6.108}$$

–

Therefore, $\delta L = 0$ for all $\delta\phi$ if and only if Poisson's equation $\nabla\cdot\epsilon\boldsymbol{E} = \rho$ is satisfied.

For $\boldsymbol{A} \to \boldsymbol{A} + \delta\boldsymbol{A}$ the same procedure gives

$$\begin{aligned} \delta L &= \iint \left(\frac{1}{\mu}\nabla\times\delta\boldsymbol{A}\cdot\boldsymbol{B} + \frac{\partial\delta\boldsymbol{A}}{\partial t}\cdot\epsilon\boldsymbol{E} - \delta\boldsymbol{A}\cdot\boldsymbol{J} \right) dV\,dt \\ &= \iint \left(\nabla\times\frac{\boldsymbol{B}}{\mu} - \frac{\partial}{\partial t}\epsilon\boldsymbol{E} - \boldsymbol{J} \right)\cdot\delta\boldsymbol{A}\, dV\,dt = 0. \end{aligned} \tag{6.109}$$

Therefore, $\delta L = 0$ for all $\delta\boldsymbol{A}$ if and only if Ampère's law $\nabla\times(\boldsymbol{B}/\mu) = \partial(\epsilon\boldsymbol{E})/\partial t + \boldsymbol{J}$ holds everywhere. Faraday's law and $\nabla\cdot\boldsymbol{B} = 0$ are automatically satisfied because of the potential representation (6.106).

Integral equations in time domain

- **Retarded potentials**

$$\mathbf{A}(\mathbf{r},t) = \mu \iint_S \frac{\mathbf{J}(\mathbf{r}', t - R/c)}{4\pi R}\, \mathrm{d}s',$$

$$\Phi(\mathbf{r},t) = -\frac{1}{\varepsilon} \iint_S \int_0^{t-R/c} \frac{\nabla' \cdot \mathbf{J}(\mathbf{r}', t')}{4\pi R}\, \mathrm{d}t' \mathrm{d}s'.$$

$$\begin{aligned} \mathrm{EFIE:} \quad & -\hat{\mathbf{n}}(\mathbf{r}) \times \left(\hat{\mathbf{n}}(\mathbf{r}) \times \partial_t \mathbf{E}^{\mathrm{inc}}(\mathbf{r},t)\right) = \\ & -\hat{\mathbf{n}}(\mathbf{r}) \times \left(\hat{\mathbf{n}}(\mathbf{r}) \times \left(\partial_t^2 \mathbf{A}(\mathbf{r},t) + \nabla \partial_t \Phi(\mathbf{r},t)\right)\right), \end{aligned}$$

$$\begin{aligned} \mathrm{MFIE:} \quad & \hat{\mathbf{n}}(\mathbf{r}) \times \partial_t \mathbf{H}^{\mathrm{inc}}(\mathbf{r},t) = \\ & \partial_t \mathbf{J}(\mathbf{r},t) - \hat{\mathbf{n}}(\mathbf{r}) \times \nabla \times \partial_t \mathbf{A}(\mathbf{r},t)/\mu, \end{aligned}$$

$$\mathrm{CFIE:} \quad \mathrm{CFIE} = \eta(1-\alpha)\,\mathrm{MFIE} + \alpha \mathrm{EFIE}.$$

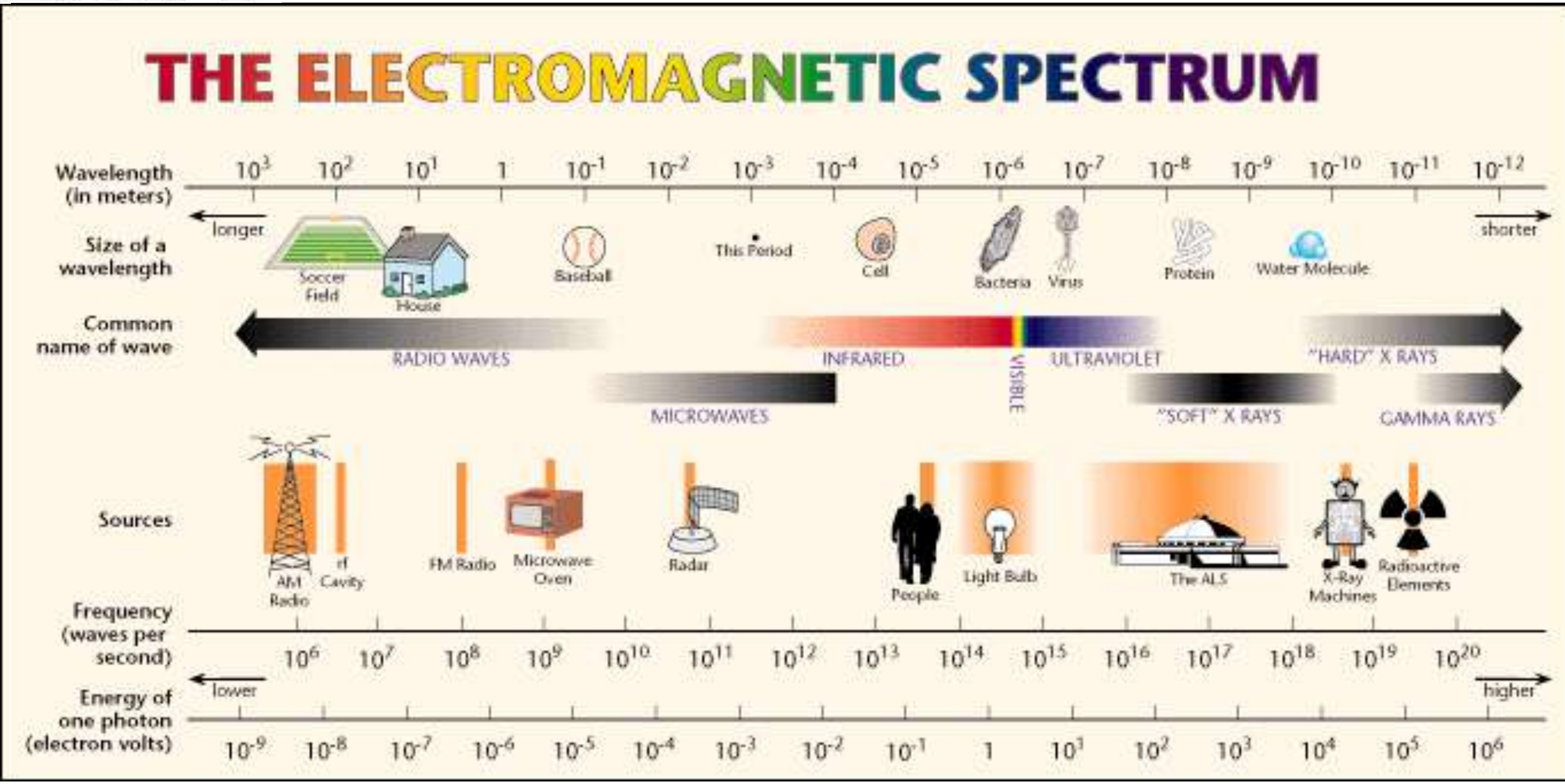
$$\mathbf{E}^{\mathrm{sca}}(\mathbf{r},t) = -\partial_t \mathbf{A}(\mathbf{r},t) - \nabla \Phi(\mathbf{r},t),$$

$$\mathbf{H}^{\mathrm{sca}}(\mathbf{r},t) = \nabla \times \mathbf{A}(\mathbf{r},t)/\mu,$$

Applications

- **Antenna**
- **Wave guides**
- **Resonant cavities**
- **Microwave heating (ovens)**
- **Wireless communication**
- **Radio, TV**
- **Cellular - mobile phones**
- **Radar**
- **Stealth objects**
- **Meta-materials**
- **Radio-navigation (marine, aero)**
- **GPS**
- **RFID**

Specrul electromagnetoc



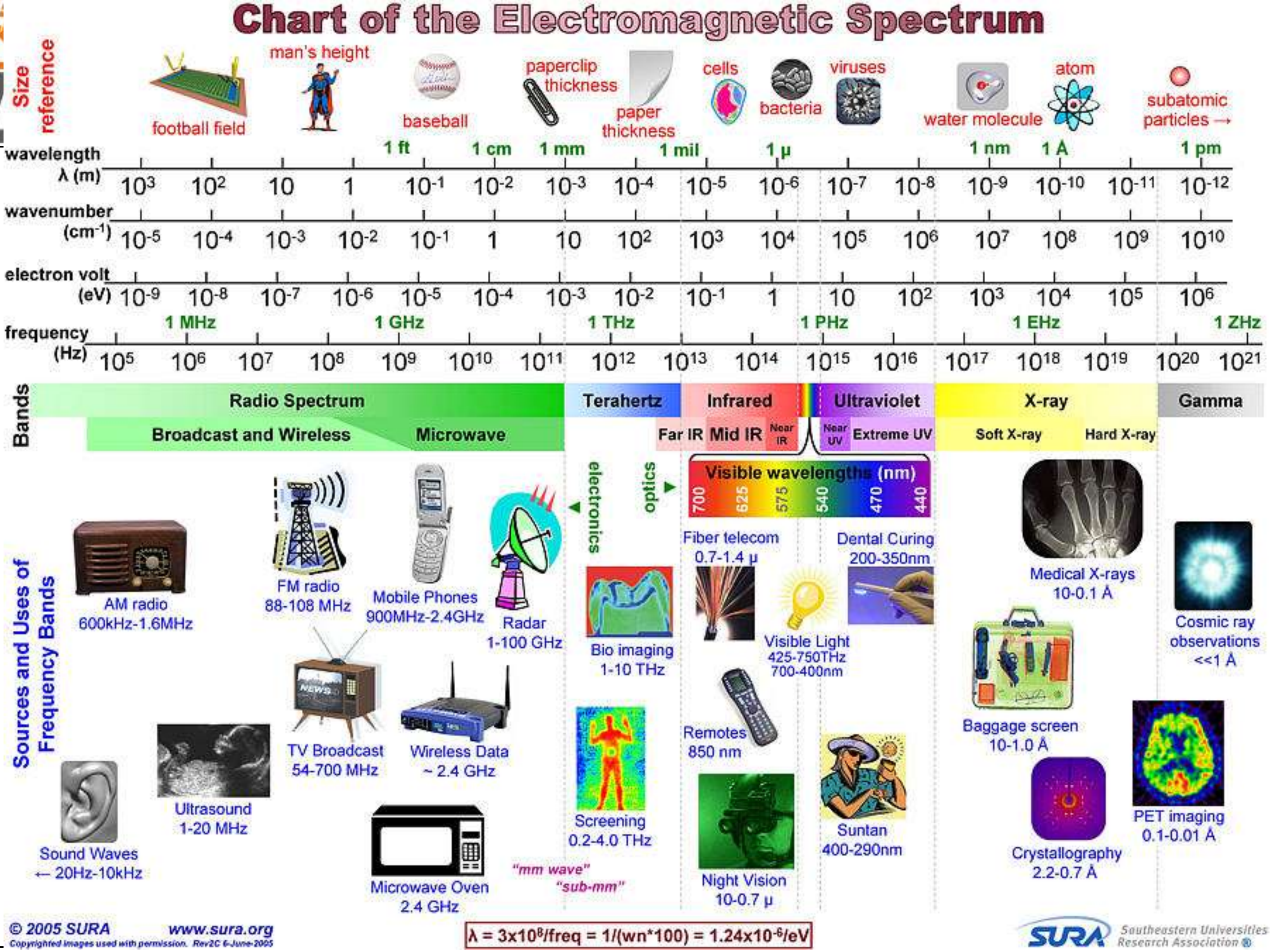


Chart of the Electromagnetic Spectrum
Size reference
football field
man's height
baseball
paperclip thickness
paper thickness
cells
bacteria
viruses
water molecule
atom
subatomic particles →
wavelength λ (m)
1 ft
1 cm
1 mm
1 mil
1 μ
1 nm
1 Å
1 pm
10^{3} 10^{2} 10 1 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10} 10^{-11} 10^{-12}
wavenumber (cm^{-1})
10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10 10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7} 10^{8} 10^{9} 10^{10}
electron volt (eV)
10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10 10^{2} 10^{3} 10^{4} 10^{5} 10^{6}
frequency (Hz)
1 MHz
1 GHz
1 THz
1 PHz
1 EHz
1 ZHz
10^{5} 10^{6} 10^{7} 10^{8} 10^{9} 10^{10} 10^{11} 10^{12} 10^{13} 10^{14} 10^{15} 10^{16} 10^{17} 10^{18} 10^{19} 10^{20} 10^{21}
Bands
Radio Spectrum
Terahertz
Infrared
Ultraviolet
X-ray
Gamma
Broadcast and Wireless
Microwave
Far IR
Mid IR
Near IR
Near UV
Extreme UV
Soft X-ray
Hard X-ray
Visible wavelengths (nm)
700 625 575 540 470 440
electronics
optics
Sources and Uses of Frequency Bands
AM radio 600kHz-1.6MHz
FM radio 88-108 MHz
Mobile Phones 900MHz-2.4GHz
Radar 1-100 GHz
Bio imaging 1-10 THz
Fiber telecom 0.7-1.4 μ
Dental Curing 200-350nm
Medical X-rays 10-0.1 Å
Cosmic ray observations <<1 Å
Visible Light 425-750THz 700-400nm
Remotes 850 nm
Baggage screen 10-1.0 Å
TV Broadcast 54-700 MHz
Wireless Data ~ 2.4 GHz
Screening 0.2-4.0 THz
Sound Waves ← 20Hz-10kHz
Ultrasound 1-20 MHz
Microwave Oven 2.4 GHz
"mm wave"
"sub-mm"
Night Vision 10-0.7 μ
Suntan 400-290nm
Crystallography 2.2-0.7 Å
PET imaging 0.1-0.01 Å
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λ = 3x10⁸/freq = 1/(wn*100) = 1.24x10⁻⁶/eV
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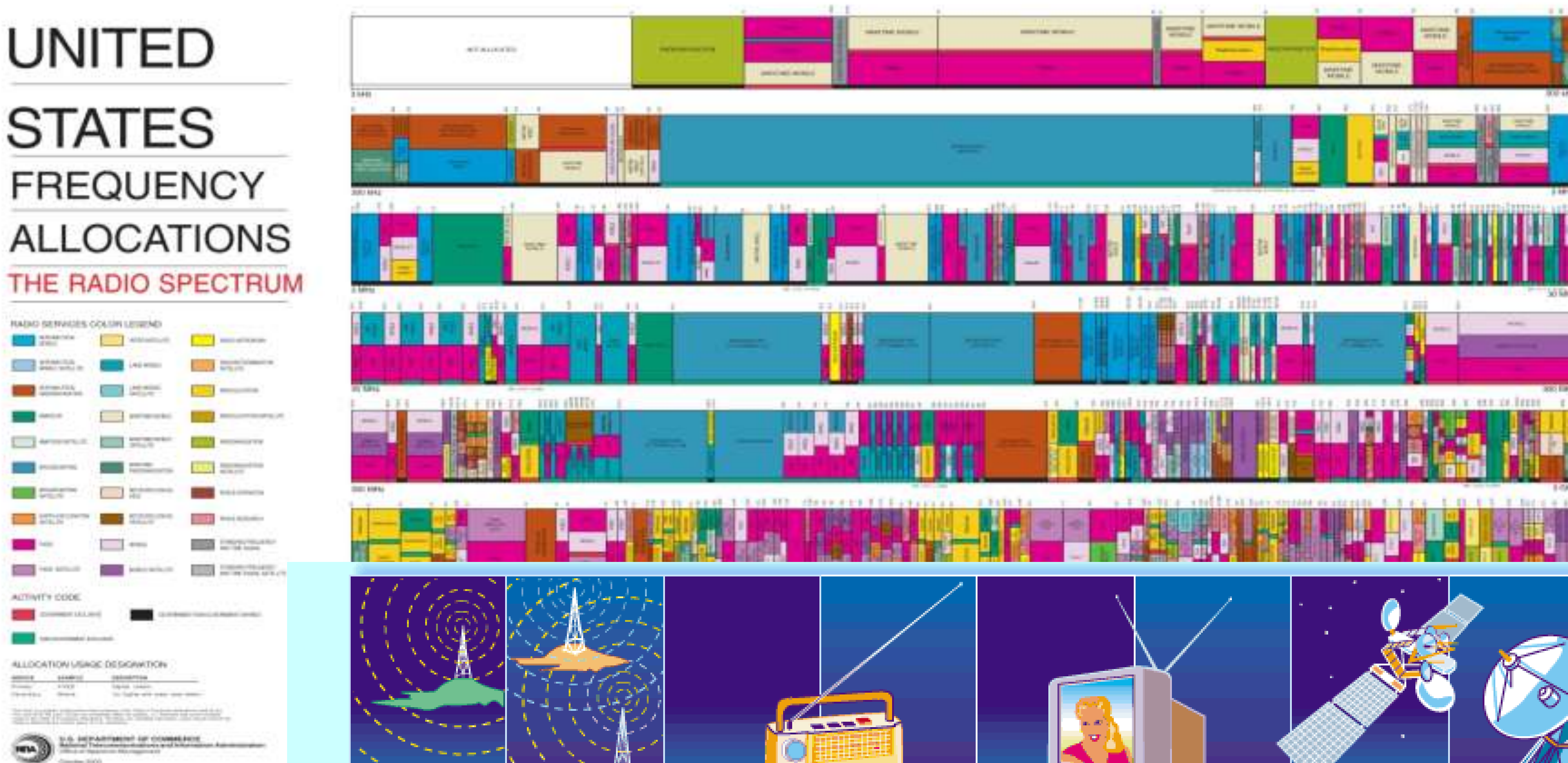
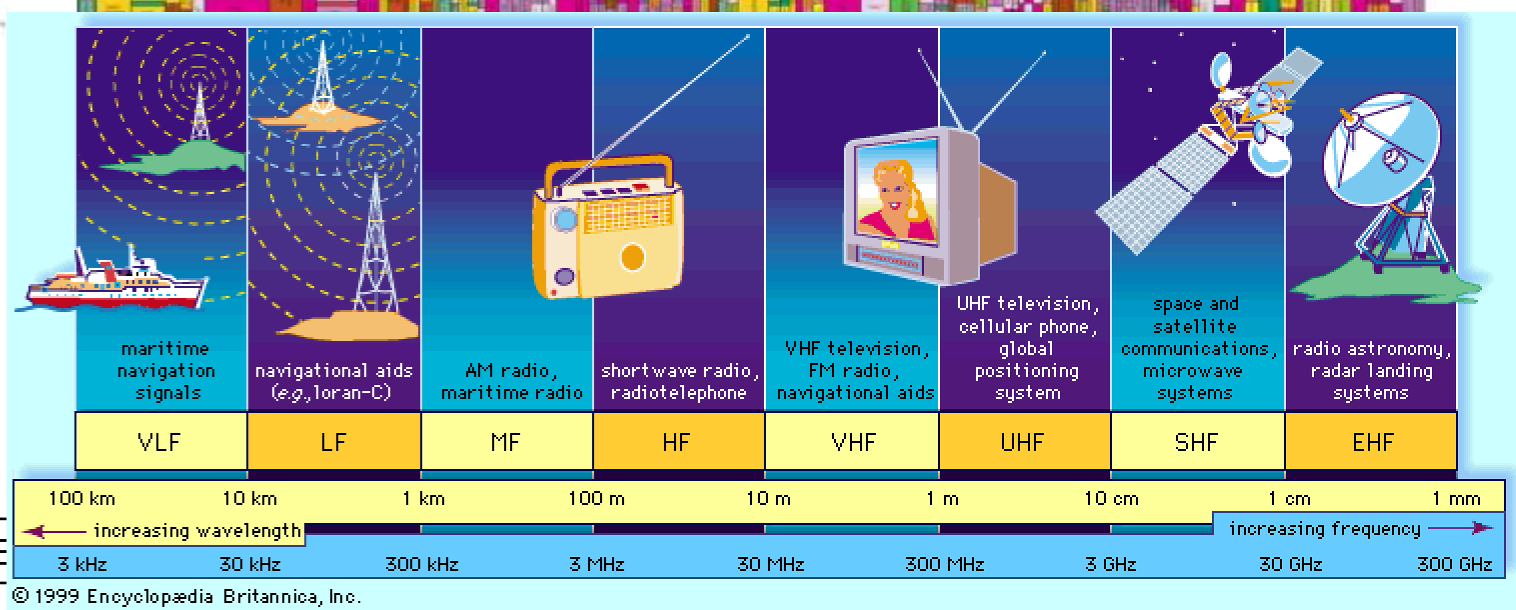




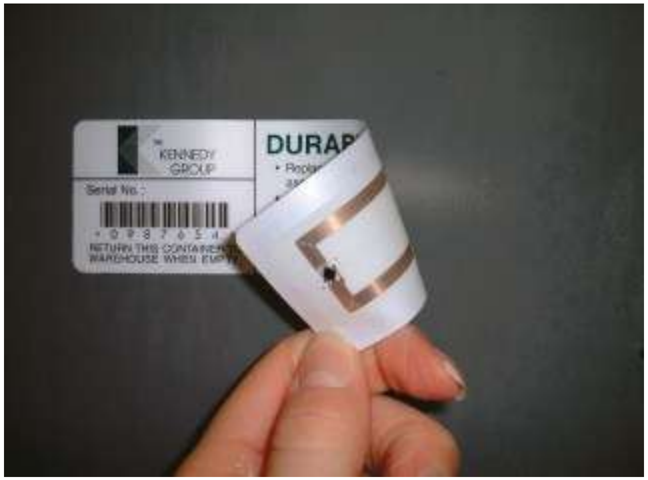
Applications

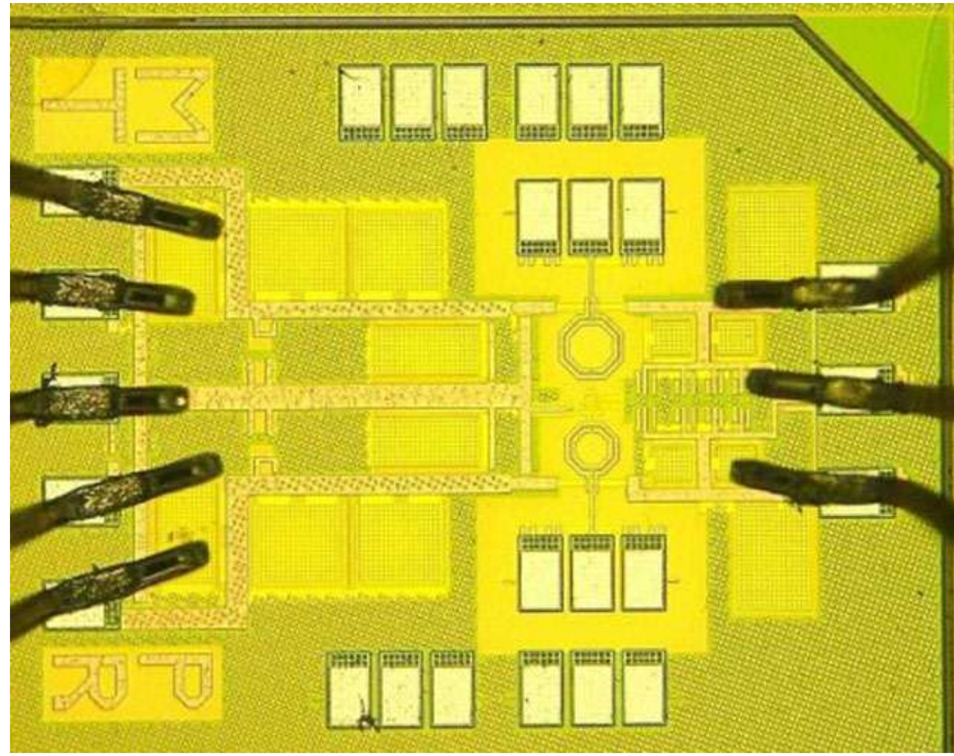
Radar

Stealth

Applications

- **RFID, RFIC**





Not so easy questions for curious people

1. How are the second order equations for FW regime?
2. What kind of potentials are used in ED regime?
3. What are the initial conditions in this regime?
4. What are boundary conditions in this regime?
5. How may be treated the open domains?
6. What is the wave scattering?
7. How is the weak formulation of FW-EM field?
8. How is the integral formulation of FW-EM field?