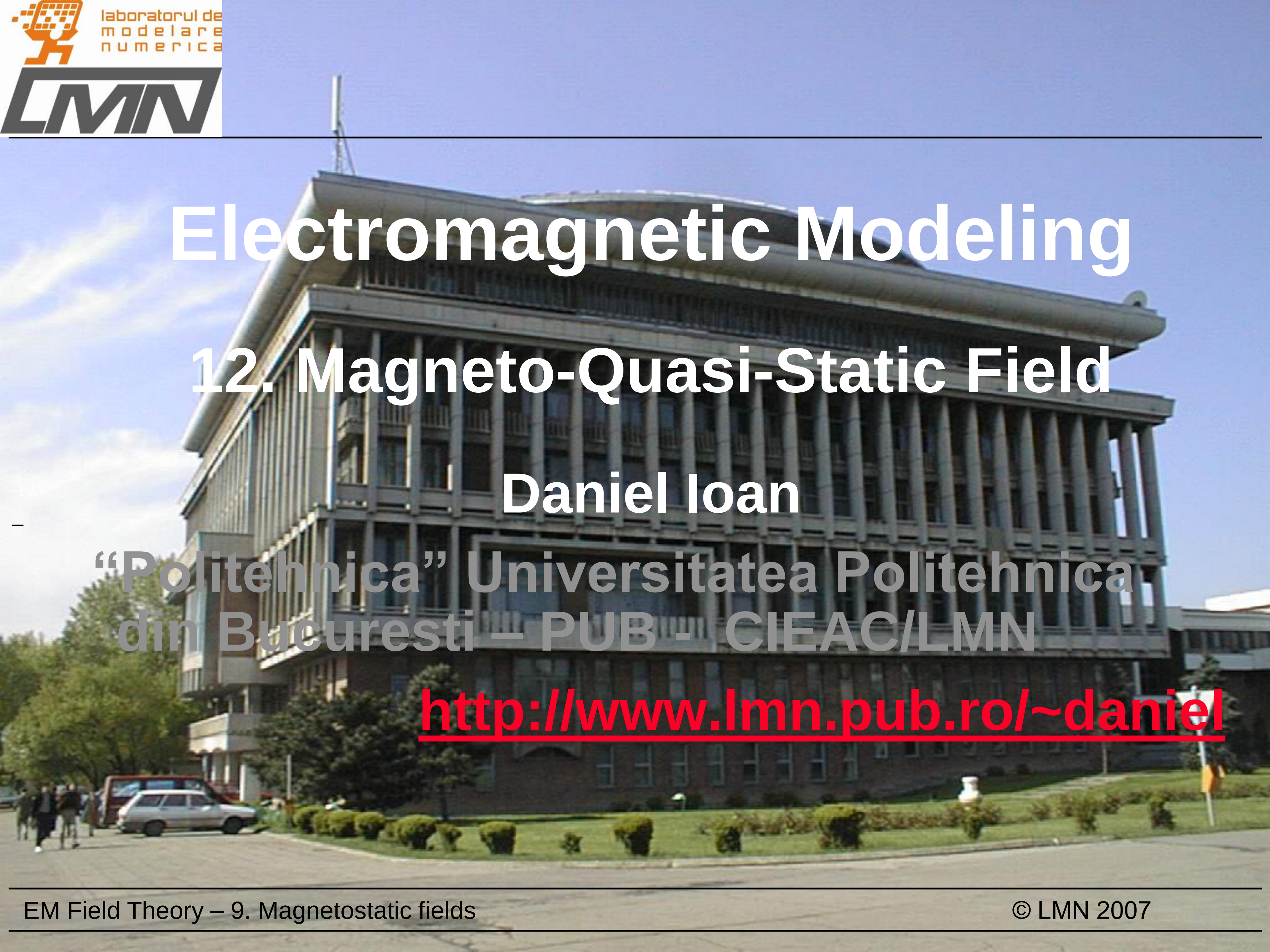

Electromagnetic Modeling

12. Magneto-Quasi-Static Field

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Contents

MQS-Magneto-Quasi-Static Field

$$1.\ \nabla \cdot \mathbf{D} = \rho$$

$$2.\ \nabla \cdot \mathbf{B} = 0$$

$$3.\ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$4.\ \nabla \times \mathbf{H} = \mathbf{J} + \cancel{\frac{\partial \mathbf{D}}{\partial t}}$$

$$5.\ \mathbf{D} = \varepsilon\ \mathbf{E} + \mathbf{P}_p(\mathbf{E})$$

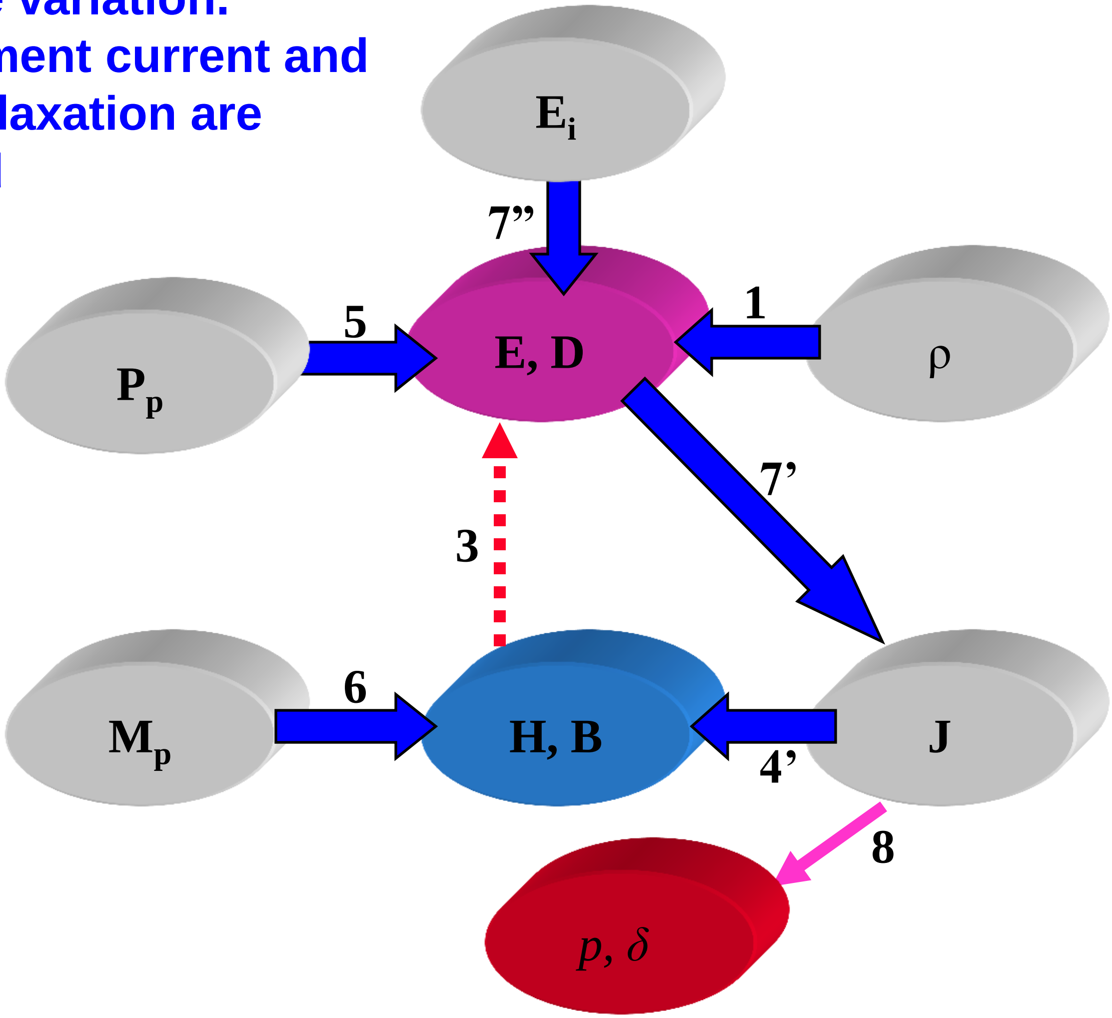
$$6.\ \mathbf{B} = \mu\ (\mathbf{H} + \mathbf{M}_p(\mathbf{H}))$$

$$7.\ \mathbf{J} = \sigma(\mathbf{E} + \mathbf{E}_i(\mathbf{E}))$$

$$8.\ p = \mathbf{E}\mathbf{J}$$

$$9.\ \nabla \cdot \mathbf{J} = \cancel{-\frac{\partial \rho}{\partial t}}$$

Slow time variation. Displacement current and charge relaxation are neglected



MQS equations

- **Hypothesis:**
  - no movement
  - slow time variation
  - capacitive effects are neglected

- **Fundamental Equations:**
- **Gauss' theorem**

$$\begin{cases} \Phi_{\Sigma} = 0 \Leftrightarrow \oint_{\Sigma} \mathbf{B} d\mathbf{A} = 0 \\ div\mathbf{B} = 0 \Rightarrow \mathbf{B} = curl\mathbf{A} \\ \mathbf{n}_{12} \cdot (\mathbf{B}_2 - \mathbf{B}_1) = 0 \Leftrightarrow div_s \mathbf{B} = 0 \end{cases}$$

- **Ampere's theorem**

$$\begin{cases} u_{m\Gamma} = i_{S_\Gamma} \Leftrightarrow \oint_{\Gamma} \mathbf{H} d\mathbf{r} = \int_{S_\Gamma} \mathbf{J} \cdot \mathbf{n} dS \\ curl\boldsymbol{H} = \boxed{\boldsymbol{J}} \Rightarrow div\boldsymbol{J} = 0 \\ \mathbf{n}_{12} \times (\mathbf{H}_2 - \mathbf{H}_1) = 0, \Leftrightarrow \mathbf{H}_{t2} = \mathbf{H}_{t1} \end{cases}$$

- **Magnetic and conductive constitutive relations**

$$\boldsymbol{B} = \boldsymbol{f}(\boldsymbol{H}) \Rightarrow \boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M}) \Rightarrow \boldsymbol{B} = \mu\boldsymbol{H} + \mu_0 \boldsymbol{M}_p$$

$$\boldsymbol{J} = g(\boldsymbol{E}) \Rightarrow \boldsymbol{J} = \sigma(\boldsymbol{E} + \boldsymbol{E}_i) \Rightarrow \boldsymbol{J} = \sigma\boldsymbol{E} + \boldsymbol{J}_i$$

- **Induction law**

$$u_{\Gamma} = -\frac{d\varphi_{S_\Gamma}}{dt} \Leftrightarrow \oint_{\Gamma} \boldsymbol{E} d\boldsymbol{r} = -\frac{d}{dt}\int_{S_\Gamma} \boldsymbol{B} \cdot \boldsymbol{n} dS \Rightarrow curl\boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \Rightarrow curl_s \boldsymbol{E} = 0$$

- **Field sources:**
  - Conduction current
- **MQS field is similar to MG+EC , but J is unknown**

MQS:	**H,E**	**B,J**	$\mathbf{J}_i$	μ, σ	$\boldsymbol{A}, V$
MG:	**H**	**B**	**J**	μ	$\boldsymbol{A}$
EC:	**E**	**J**	-	σ	V

MQS equations – AV formulation

- **First order:**

$$\begin{cases} div\boldsymbol{B}=0 \\ rot\boldsymbol{H}=\boldsymbol{J}\Rightarrow div\boldsymbol{J}=0 \\ rot\boldsymbol{E}=-\dfrac{\partial\boldsymbol{B}}{\partial t} \\ \boldsymbol{B}=\mu\boldsymbol{H}+\boldsymbol{B}_r \\ \boldsymbol{J}=\sigma\boldsymbol{E}+\boldsymbol{J}_i \end{cases}$$

- **On surfaces:**

$$\begin{cases} div_s\boldsymbol{B}=0\Rightarrow B_{n1}=B_{n2};div_s\boldsymbol{A}=0\Rightarrow \boxed{A_{n1}=A_{n2}} \\ rot_s\boldsymbol{H}=\boldsymbol{J}_s\Leftrightarrow \boldsymbol{H}_{t2}=\boldsymbol{H}_{t1};rot_s\boldsymbol{A}=0\Rightarrow \boxed{\boldsymbol{A}_{t2}=\boldsymbol{A}_{t1}} \\ \Rightarrow \boxed{\boldsymbol{n}\times\nu_2 rot\boldsymbol{A}_2=\boldsymbol{n}\times\nu_1 rot\boldsymbol{A}_1}\Leftrightarrow rot_s(\nu rot\boldsymbol{A})=0 \\ rot_s\boldsymbol{E}=0\Leftrightarrow \boldsymbol{E}_{t2}=\boldsymbol{E}_{t1}\Rightarrow \boxed{V_1=V_2} \\ div_s\boldsymbol{J}=0\Rightarrow J_{n1}=J_{n2}\Rightarrow \boxed{\sigma_1 dV_1/dn=\sigma_2 dV_2/dn} \end{cases}$$

- **Second order:** $div\boldsymbol{B}=0\Rightarrow \boldsymbol{B}=rot\boldsymbol{A};\quad div\boldsymbol{A}=0?$ **Coulomb g.c.**

$$\begin{cases} rot\boldsymbol{H}=\boldsymbol{J}\Rightarrow rot\,\nu(\boldsymbol{B}-\boldsymbol{B}_r)=\sigma\boldsymbol{E}+\boldsymbol{J}_i\Rightarrow rot\,\nu\boldsymbol{B}=rot(\nu\boldsymbol{B}_r)+\sigma\boldsymbol{E}+\boldsymbol{J}_i \\ rot\boldsymbol{E}=-\dfrac{\partial\boldsymbol{B}}{\partial t}\Rightarrow rot\boldsymbol{E}+rot\dfrac{\partial\boldsymbol{A}}{\partial t}=0\Rightarrow \boldsymbol{E}+\sigma\dfrac{\partial\boldsymbol{A}}{\partial t}=-gradV \\ \boldsymbol{B}=\mu\boldsymbol{H}+\boldsymbol{B}_r\Rightarrow \boldsymbol{H}=\nu(\boldsymbol{B}-\boldsymbol{B}_r) \\ \boldsymbol{J}=\sigma\boldsymbol{E}+\boldsymbol{J}_i\Rightarrow div(\sigma\boldsymbol{E})=0\Rightarrow \boxed{div(\sigma gradV)+div(\sigma\dfrac{\partial\boldsymbol{A}}{\partial t})=0\Rightarrow \Delta V=0} \end{cases}$$

$\Rightarrow$

Parabolic equations!

$$rot(\nu rot\boldsymbol{A})=rot(\nu\boldsymbol{B}_r)+\sigma\boldsymbol{E}+\boldsymbol{J}_i\Rightarrow \boxed{rot(\nu rot\boldsymbol{A})+\sigma\left(\frac{\partial\boldsymbol{A}}{\partial t}+gradV\right)=rot(\nu\boldsymbol{B}_r)+\boldsymbol{J}_i}$$

MQS equations - TVm (TΩ) formulation

- **First order:**

$$\begin{cases} div\mathbf{B} = 0; rot\mathbf{H} = \mathbf{J} \\ rot\mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \\ \mathbf{B} = \mu\mathbf{H} + \mathbf{B}_r; \\ \mathbf{J} = \sigma\mathbf{E} + \mathbf{J}_i \end{cases}$$

- **On surfaces:**

$$\begin{cases} div_s\mathbf{T} = 0 \Rightarrow \boxed{T_{n1} = T_{n2}; \\ rot_s\mathbf{T} = \mathbf{J}_s \Rightarrow \mathbf{T}_{t2} = \mathbf{T}_{t1};} \; rot_s(\mathbf{E}) = \boxed{rot_s(\rho rot\mathbf{T}) = 0} \\ \mathbf{H} = \mathbf{T} - gradV_m \Rightarrow \\ \boxed{V_{m1} = V_{m2}; \mu_2 dV_{m2}/dn = \mu_2 dV_{m2}/dn} \end{cases}$$

- **Second order:**

$$\begin{cases} div\mathbf{B} = 0 \Rightarrow div(\mu\mathbf{H} + \mathbf{B}_r) = 0 \Rightarrow div(\mu\mathbf{T} - \mu gradV_m + \mathbf{B}_r) = 0 \Rightarrow \boxed{\Delta V_m = -\nu div\mathbf{B}_r} \\ rot\mathbf{H} = \mathbf{J} \Rightarrow div\mathbf{J} = 0 \Rightarrow \mathbf{J} = rot\mathbf{T}; div\mathbf{T} = 0 \Rightarrow \\ rot(\mathbf{H} - \mathbf{T}) = 0 \Rightarrow \mathbf{H} - \mathbf{T} = -gradV_m \Rightarrow \mathbf{H} = \mathbf{T} - gradV_m \\ rot\mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \Rightarrow rot\left(\rho(rot\mathbf{T} - \mathbf{J}_i)\right) = \dfrac{\partial \mathbf{B}}{\partial t} \qquad \Rightarrow \\ \mathbf{B} = \mu\mathbf{H} + \mathbf{B}_r \Rightarrow rot\left(\rho(rot\mathbf{T} - \mathbf{J}_i)\right) = \mu\dfrac{\partial \mathbf{H}}{\partial t} \\ \mathbf{J} = \sigma\mathbf{E} + \mathbf{J}_i \Rightarrow \mathbf{E} = \rho(\mathbf{J} - \mathbf{J}_i) \Rightarrow \mathbf{E} = \rho(rot\mathbf{T} - \mathbf{J}_i) \end{cases}$$

$$rot\left(\rho(rot\mathbf{T} - \mathbf{J}_i)\right) = \mu\frac{\partial \mathbf{H}}{\partial t} \Rightarrow \boxed{rot\left(\rho rot\mathbf{T}\right) - \mu\frac{\partial \mathbf{T}}{\partial t} + \mu grad\frac{\partial V_m}{\partial t} = rot\left(\rho\mathbf{J}_i\right)}$$

Boundary conditions for AV formuation

- **Magnetic vector potential (parabolic, gen. diffusion vect. eq.):**

$$rot(\nu rot\boldsymbol{A}) + \boxed{grad(\nu div\boldsymbol{A})} + \sigma\left(\frac{\partial\boldsymbol{A}}{\partial t} + gradV\right) = rot(\nu\boldsymbol{B}_r) + \boldsymbol{J}_i$$

- **Electric scalar potential (gen. Laplace eq.):** **with Coulomb gauge condition**

$$div(\sigma gradV) + div\left(\sigma\frac{\partial\boldsymbol{A}}{\partial t}\right) = 0; div(\sigma\boldsymbol{A}) = 0 \Rightarrow \boxed{div(\sigma gradV) = 0}$$

$$\sigma = ct \Rightarrow div\boldsymbol{A} = 0; \boxed{\Delta V = 0,} \text{with } V = f(P) \text{ on } S_D; \; dV/dn = j_n(P) \text{ on } S_N$$

– **Vector boundary conditions are necessary for a unique solution**

Dirichlet b.c. **Neumann b,c.**

$$\boxed{\boldsymbol{A}_t(P) = \boldsymbol{f}_{DA}(P),} \text{on } S_B \neq \emptyset$$

$$\boxed{\nu\boldsymbol{n}\times(\text{curl}\boldsymbol{A}\times\boldsymbol{n}) = \boldsymbol{f}_{NA}(P) = \boldsymbol{h}_t,} \text{ on } S_H = \Sigma - S_B$$

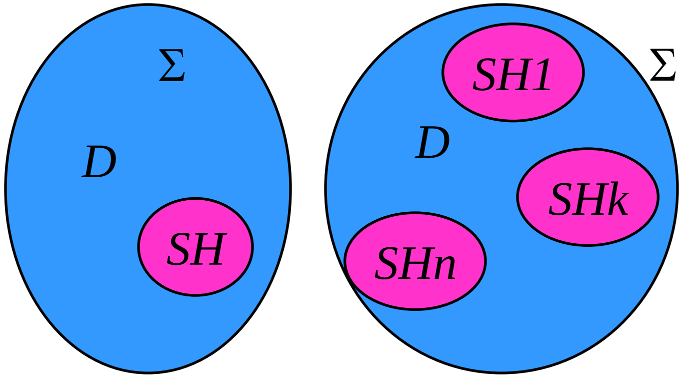
For a unique vector potential A:

$$\boxed{n\cdot\boldsymbol{A} = A_n(P), on S_H}$$

If

$$S_H = \bigcup_{k=1}^{m} S_k \quad \text{multipleconnex} \Rightarrow \boxed{\int_{S_k}\boldsymbol{B}\cdot d\boldsymbol{S} = \varphi_k \text{ or } \int_{C_{km}}\boldsymbol{H}\cdot d\boldsymbol{r} = u_k}; k = 1,\ldots, m-1$$

The fundamental MQS problem in terms of fields



Input (known) data:

- Computational domain *D* bounded by Σ
- (CM) Material characteristics σ,μ(**r**)>0 in *D*
- (CD) Internal field sources Ji(r) in *D*
- (CΣ') Boundary conditions

$\mathbf{E}_t(\mathbf{r})$ on *SD* connected and $J_n(\mathbf{r})$ on *SN=Σ-SD*

$\mathbf{H}_t(\mathbf{r})$ on *SH* connected and $B_n(\mathbf{r})$ on *SB=Σ-SH*

- (IC) Initial conditions: **B(r)** on *D*

For non-connected Dirichlet surfaces $S_H = \bigcup_{k=1}^{n} S_{Hk}, S_{Hk} \cap S_{Hj} = \emptyset$

in addition to (CΣ') solution uniqueness requires :

(CΣ'') $\boxed{U_k = \int_{PkP_0} \mathbf{H}_t d\mathbf{r} \text{ or } \Phi_k = \int_{S_{Fk}} \mathbf{B}_n dS,}$ for $k = 1,2,.., n\text{-}1, \text{and } U_n = 0.$

Output data (solution): **H(r), B(r), J(r), E(r),** in D

For unique potential A,V: $n \cdot \mathbf{A} = A_n(P), on S_H ; V = f_D(P) on S_D$

and gauged equation: ….. $grad(\nu div \mathbf{A}) \Rightarrow div \mathbf{A} = 0$

Eddy current problems: MG+MQS

- **In most practical cases:**

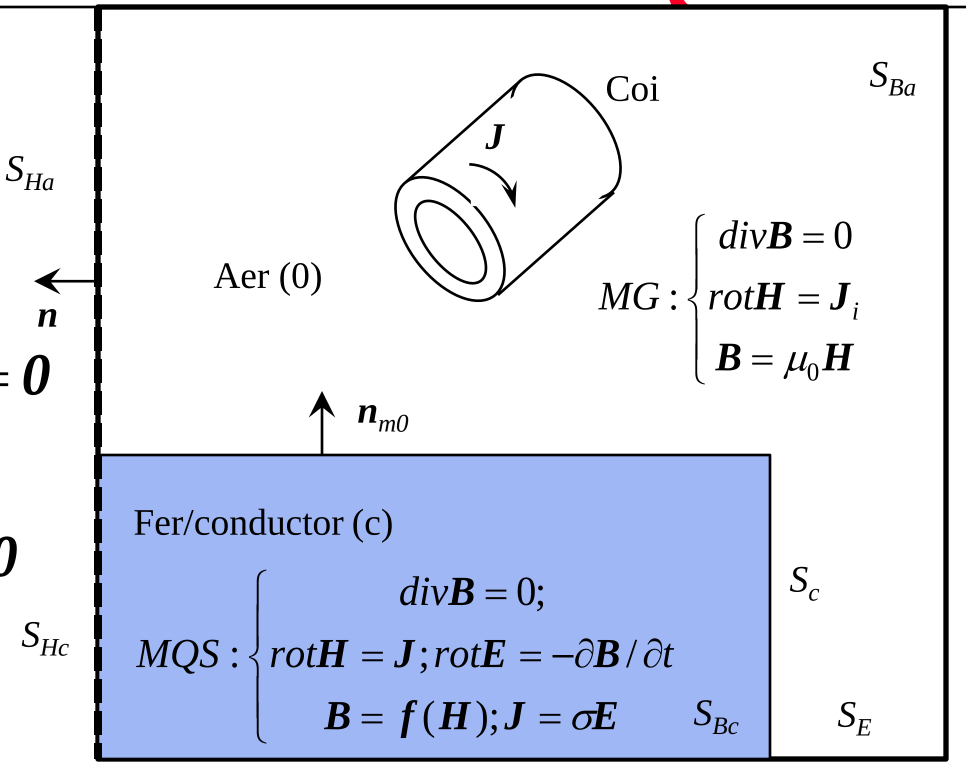
initial cond. $\boldsymbol{B}(0)=0$

boundary conditions

$$S_H: \quad \boldsymbol{H}_t=\boldsymbol{n}\times(\boldsymbol{H}\times\boldsymbol{n})=\boldsymbol{0}\Rightarrow\boldsymbol{n}\times\boldsymbol{H}=\boldsymbol{0}$$

$$S_B: \quad B_n=\boldsymbol{n}\cdot\boldsymbol{B}=-b(\boldsymbol{r})\Rightarrow\boldsymbol{n}\cdot\boldsymbol{B}=0$$

$$S_E: \quad \boldsymbol{E}_t=\boldsymbol{n}\times(\boldsymbol{E}\times\boldsymbol{n})=\boldsymbol{0}\Rightarrow\boldsymbol{n}\times\boldsymbol{E}=\boldsymbol{0}$$



interface conditions

$$S_c: \quad \nabla_s\times\boldsymbol{H}=\boldsymbol{n}_{12}\times(\boldsymbol{H}_2-\boldsymbol{H}_1)=\boldsymbol{0}\Rightarrow\boldsymbol{n}_{ca}\times\boldsymbol{H}_a=\boldsymbol{n}_{ca}\times\boldsymbol{H}_c\Rightarrow\boldsymbol{H}_{ta}=\boldsymbol{H}_{tc}$$

$$\nabla_s\cdot\boldsymbol{B}=\boldsymbol{n}_{12}\cdot(\boldsymbol{B}_2-\boldsymbol{B}_1)=0\Rightarrow\boldsymbol{n}_{ca}\cdot\boldsymbol{B}_a=\boldsymbol{n}_{ca}\cdot\boldsymbol{B}_c\Rightarrow B_{na}=B_{nc}$$

$$\nabla_s\cdot\boldsymbol{J}=\boldsymbol{n}_{12}\cdot(\boldsymbol{J}_2-\boldsymbol{J}_1)=0\Rightarrow 0=\boldsymbol{n}_{ca}\cdot\boldsymbol{J}_a=\boldsymbol{n}_{ca}\cdot\boldsymbol{J}_c\Rightarrow J_{nc}=0$$

$$\nabla_s\times\boldsymbol{E}=\boldsymbol{n}_{12}\times(\boldsymbol{E}_2-\boldsymbol{E}_1)=\boldsymbol{0}\Rightarrow\boldsymbol{n}_{ca}\times\boldsymbol{E}_a=\boldsymbol{n}_{ca}\times\boldsymbol{E}_c\Rightarrow\boldsymbol{E}_{ta}=\boldsymbol{E}_{tc}$$

Semnificatia conditiilor de frontiera

S_{Ha} : $\mathbf{H} \times \mathbf{n} = \mathbf{J}_s$, dar de obicei frontiera nu este pânză de curent şi S_{Hc} : $\mathbf{H} \times \mathbf{n} = 0$,

condiţie îndeplinită pe suprafeţele de simetrie pe care liniile de câmp pică perpendicular sau la suprafaţa domeniilor feromagnetice ideale;

S_B : $\mathbf{n} \cdot \mathbf{B} = b$, dar de obicei S_B : $\mathbf{n} \cdot \mathbf{B} = 0$, condiţie îndeplinită pe suprafeţele de simetrie pe care liniile de câmp se preling fără să le traverseze, sau pe suprafeţele aflate la depărtare suficient de mare, ca valoarea componentei normale a inducţiei să se anuleze.

S_{Hc} : $\mathbf{H} \times \mathbf{n} = 0$, condiţie îndeplinită pe suprafeţele de simetrie pe care liniile de câmp pică perpendicular sau la suprafaţa domeniilor feromagnetice ideale;

S_E : $\mathbf{n} \times \mathbf{E} = 0$, condiţie îndeplinită pe suprafeţele de simetrie sau nu, care sunt echipotenţiale electric (de exemplu, pe electrozii de potenţial cunoscut), pe care liniile de câmp electric pică perpendicular.

Consecinţă a acestor condiţii, rezultă: S_{Hc} : $\mathbf{n} \cdot \mathbf{J} = 0$; S_E : $\mathbf{n} \cdot \mathbf{B} = 0$, ceea ce înseamnă că S_{Hc}este marginea conductorului spre un dielectric, iar S_E nu este traversată de linii ale câmpului magnetic.

Second order equations:

$$D_a : \begin{cases} \mathbf{B} = \nabla \times \mathbf{A} \\ \nabla \times (\nu_0 \nabla \times \mathbf{A}) - \nabla \cdot (\nu_0 \nabla \cdot \mathbf{A}) = \mathbf{J}_0 \quad ; \\ \mathbf{H} = \nu_0 \nabla \times \mathbf{A} \end{cases}$$

$$D_c : \begin{cases} \mathbf{B} = \nabla \times \mathbf{A} \\ \nabla \times (\nu \nabla \times \mathbf{A}) - \nabla \cdot (\nu_0 \nabla \cdot \mathbf{A}) = \mathbf{J} - \nabla \times \mathbf{I} \\ \mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \\ \mathbf{H} = \nu \mathbf{B} + \mathbf{I} \\ \mathbf{J} = \sigma \mathbf{E} \end{cases} \qquad (2.60)$$

Din care, după eliminarea densităţii de curent şi a câmpului electric, se obţin ecuaţiile:

$$D_c : \nabla \times (\nu \nabla \times \mathbf{A}) - \nabla \cdot (\nu_0 \nabla \cdot \mathbf{A}) + \sigma(\nabla V + \frac{\partial \mathbf{A}}{\partial t}) + \nabla \times \mathbf{I} = 0;$$
$$\nabla \cdot \left(\sigma \left(\nabla V + \frac{\partial \mathbf{A}}{\partial t} \right) \right) = 0, \qquad (2.61)$$

satisfăcute de **A**-V în D_c, la care se adaugă ecuaţia potenţialului **A** din D_a:

$$\nabla \times (\nu_0 \nabla \times \mathbf{A}) - \nabla \cdot (\nu_0 \nabla \cdot \mathbf{A}) = \mathbf{J}_0. \qquad (2.62)$$

Boundary conditions for potentials:

$$S_{Ha} : \nu_0(\nabla \times \mathbf{A}) \times \mathbf{n} = \mathbf{J}_s; \qquad \mathbf{n} \cdot \mathbf{A} = 0; \tag{2.63}$$

$$S_B : \mathbf{n} \times \mathbf{A} = \alpha; \;\; \text{dar de obicei} \;\; S_B : \mathbf{n} \times \mathbf{A} = 0 \tag{2.64}$$

$$S_{Hc} : (\nu\nabla \times \mathbf{A} + \mathbf{I}) \times \mathbf{n} = 0; \qquad \mathbf{n} \cdot \left(\nabla V + \frac{\partial \mathbf{A}}{\partial t}\right) = 0 \tag{2.65}$$

$$S_E : \mathbf{n} \times \left(\nabla V + \frac{\partial \mathbf{A}}{\partial t}\right) = 0; \qquad \mathbf{n} \times \mathbf{A} = 0; \qquad V = V_0; \tag{2.66}$$

$$\begin{aligned} S_{ac} &: \mathbf{n}_c \cdot (\nabla \times \mathbf{A}_a - \nabla \times \mathbf{A}_c) = 0; \\ &\mathbf{n}_c \times (\nu_0(\nabla \times \mathbf{A}_a) - \nu\nabla \times \mathbf{A}_c + \mathbf{I}) = 0; \\ &\mathbf{n}_c \cdot \left(\nabla V + \frac{\partial \mathbf{A}_c}{\partial t}\right) = 0. \end{aligned} \tag{2.67}$$

Weak form of gauged field

Forma slabă a acestor ecuaţii, în care potenţialul vector satisface implicit condiţia de etalonare Coulomb este:

$$
\begin{aligned}
&\int_{D_a} \left(\nu_0(\nabla\times\mathbf{W})\cdot(\nabla\times\mathbf{A}) + \nu_0\nabla\cdot\mathbf{W}\nabla\cdot\mathbf{A}\right) \mathrm{d}\Omega + \\
&+ \int_{D_c} \left(\nu(\nabla\times\mathbf{W})\cdot(\nabla\times\mathbf{A}) + \nu\nabla\cdot\mathbf{W}\nabla\cdot\mathbf{A} + \sigma\mathbf{W}\left(\nabla V + \frac{\partial\mathbf{A}}{\partial t}\right)\right) \mathrm{d}\Omega = \\
&\int_{Da} \mathbf{W}\cdot\mathbf{J}_0 \mathrm{d}\Omega - \int_{D_c} (\nabla\times\mathbf{W})\cdot\mathbf{I}\mathrm{d}\Omega + \int_{S_{Ha}} \mathbf{W}\cdot\mathbf{J}_s \mathrm{d}S \\
- \quad &\int_{D_c} \sigma\nabla N\cdot\left(\nabla V + \frac{\partial\mathbf{A}}{\partial t}\right) \mathrm{d}\Omega = 0
\end{aligned}
\tag{2.68}
$$

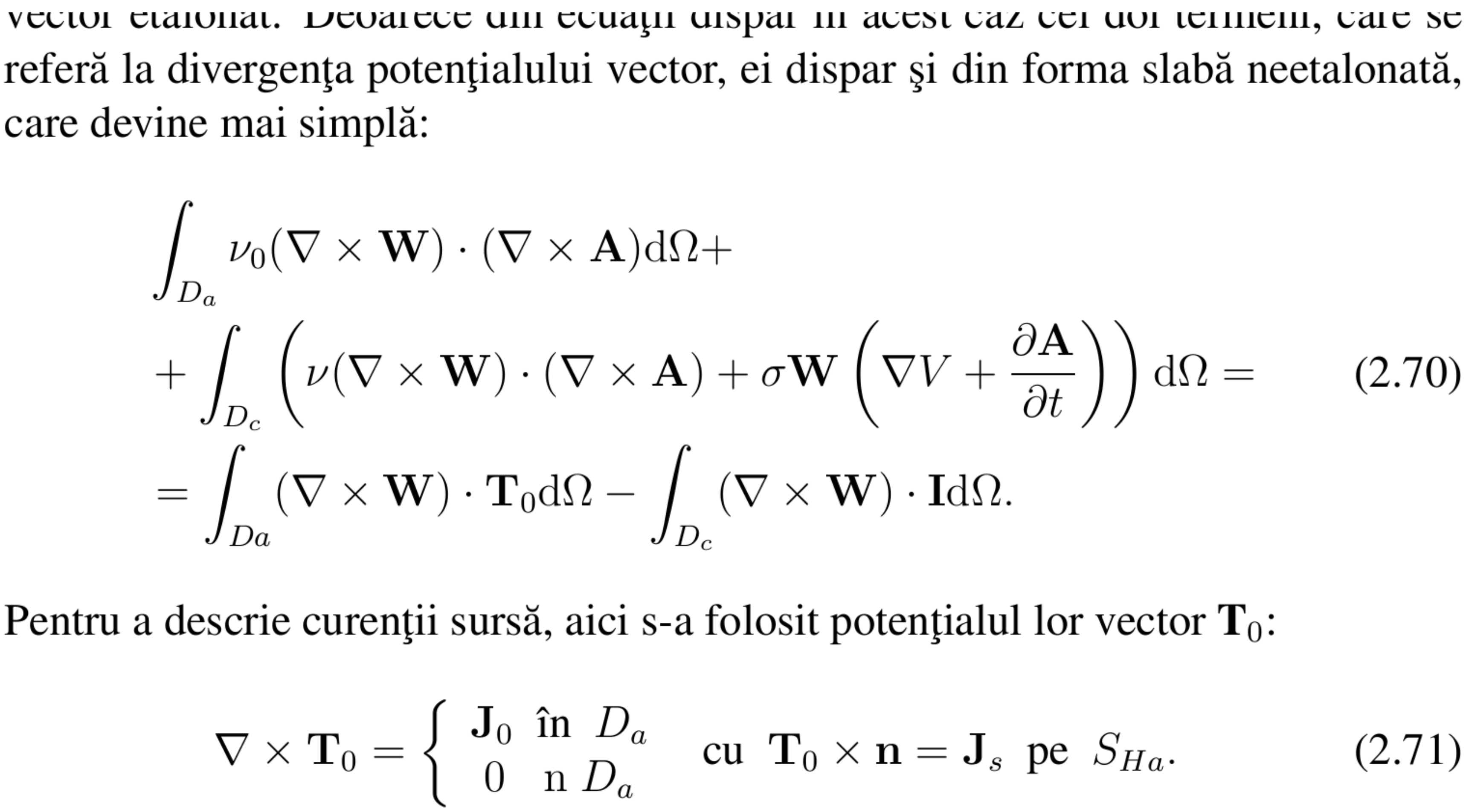
Aici s-a notat cu **W** o funcţie arbitrară de test vectorială şi cu N o funcţie arbitrară de test scalară, dar care satisfac următoarele condiţii esenţiale de frontieră:

$$
S_{Hc}\bigcup S_{Ha} : \mathbf{n}\cdot\mathbf{W} = 0; \quad S_E\bigcup S_B : \mathbf{n}\times\mathbf{W} = 0; \quad S_E : N = 0. \tag{2.69}
$$

Dacă domeniul conductor are conductivitatea σ constantă, atunci potenţialul scalar V poate fi ales nul, iar soluţia problemei este dată doar de potenţialul vector **A,** definit în întregul domeniu de calcul.

There is not an energy functional!!!!

Weak form of un-gauged field



vector etalonat. Deoarece din ecuaţii dispar în acest caz cei doi termeni, care se referă la divergenţa potenţialului vector, ei dispar şi din forma slabă neetalonată, care devine mai simplă:

$$\int_{D_a} \nu_0 (\nabla \times \mathbf{W}) \cdot (\nabla \times \mathbf{A}) \mathrm{d}\Omega +$$
$$+ \int_{D_c} \left(\nu (\nabla \times \mathbf{W}) \cdot (\nabla \times \mathbf{A}) + \sigma \mathbf{W} \left(\nabla V + \frac{\partial \mathbf{A}}{\partial t} \right) \right) \mathrm{d}\Omega = \tag{2.70}$$
$$= \int_{Da} (\nabla \times \mathbf{W}) \cdot \mathbf{T}_0 \mathrm{d}\Omega - \int_{D_c} (\nabla \times \mathbf{W}) \cdot \mathbf{I} \mathrm{d}\Omega.$$

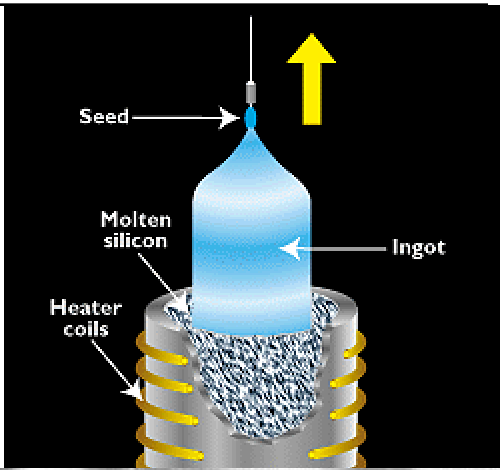
Pentru a descrie curenţii sursă, aici s-a folosit potenţialul lor vector $\mathbf{T}_0$:

$$\nabla \times \mathbf{T}_0 = \begin{cases} \mathbf{J}_0 & \text{în } D_a \\ 0 & \text{n } D_a \end{cases} \quad \text{cu } \mathbf{T}_0 \times \mathbf{n} = \mathbf{J}_s \text{ pe } S_{Ha}. \tag{2.71}$$

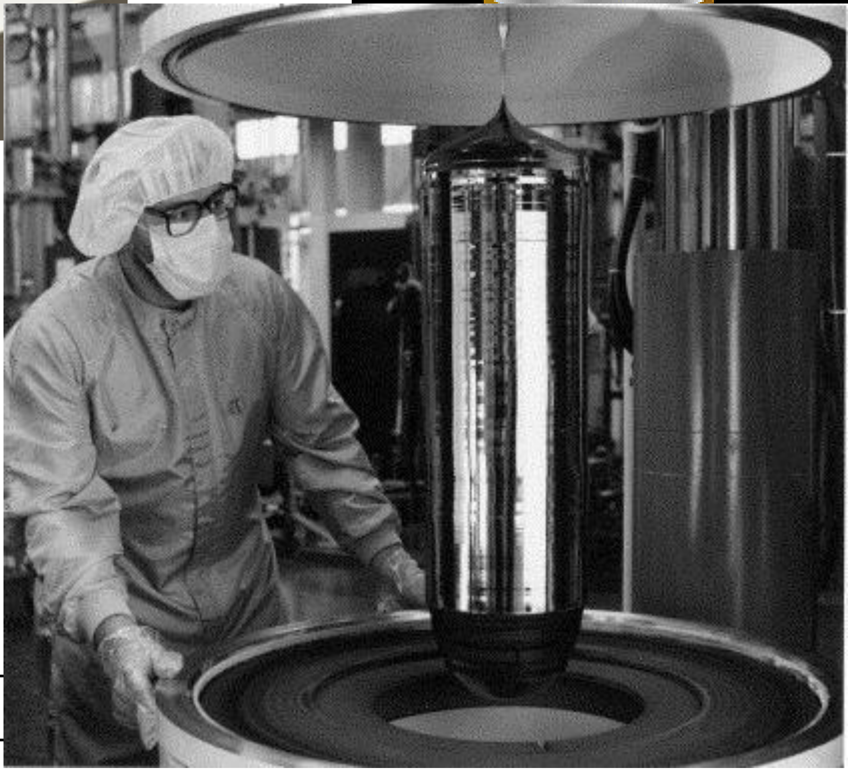
MQS applications

- Transformers
- Induction motors
- Electric generators
- Eddy currents breaks
- Inductive heating
- Meters – inductions counters
- Eddy currents testing

Inductive heating

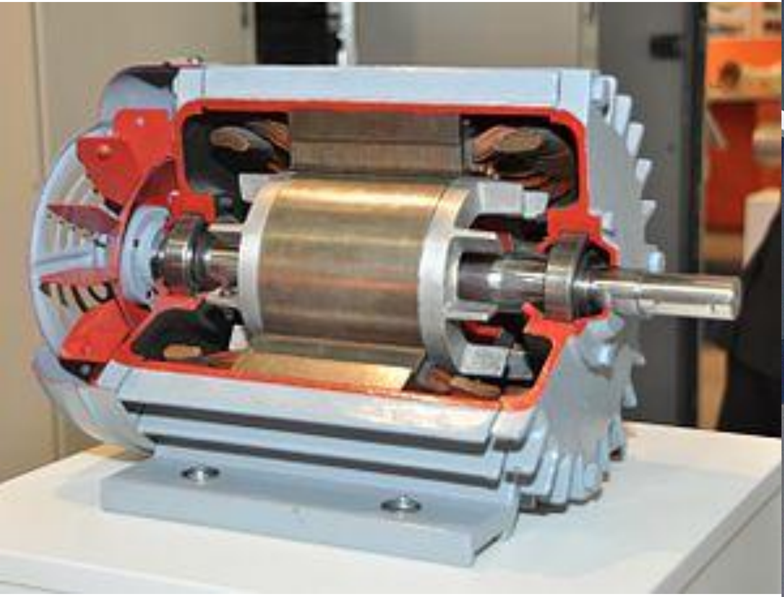


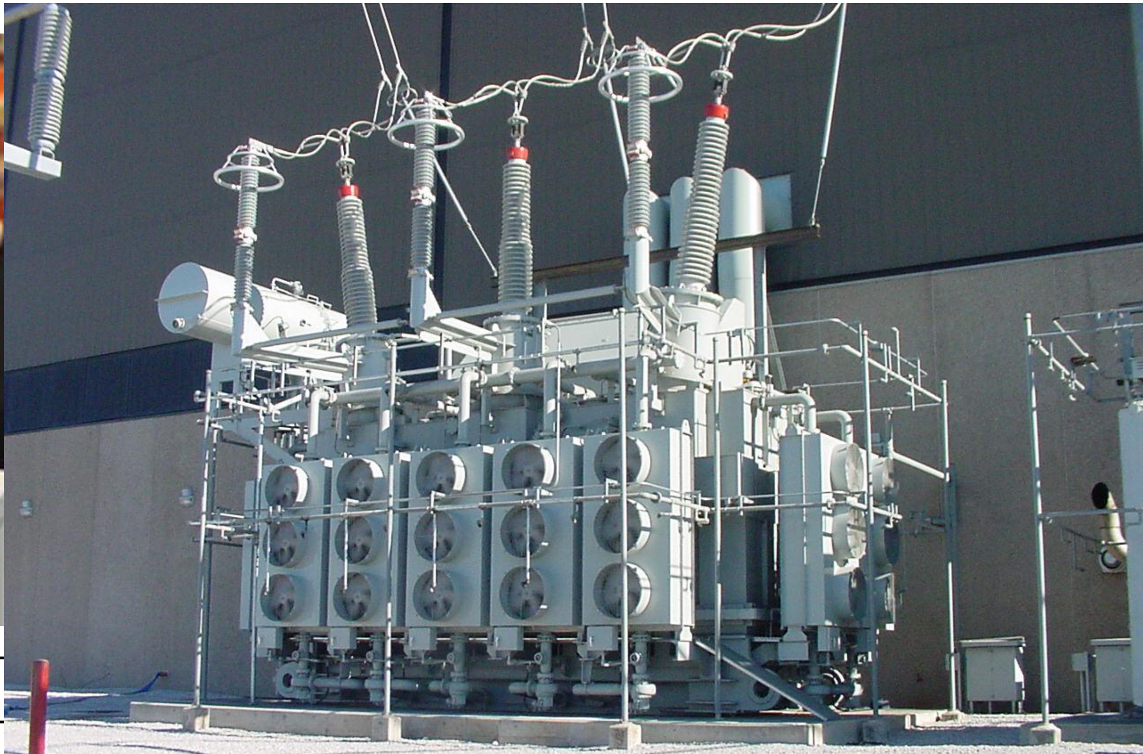
Induction cooktop



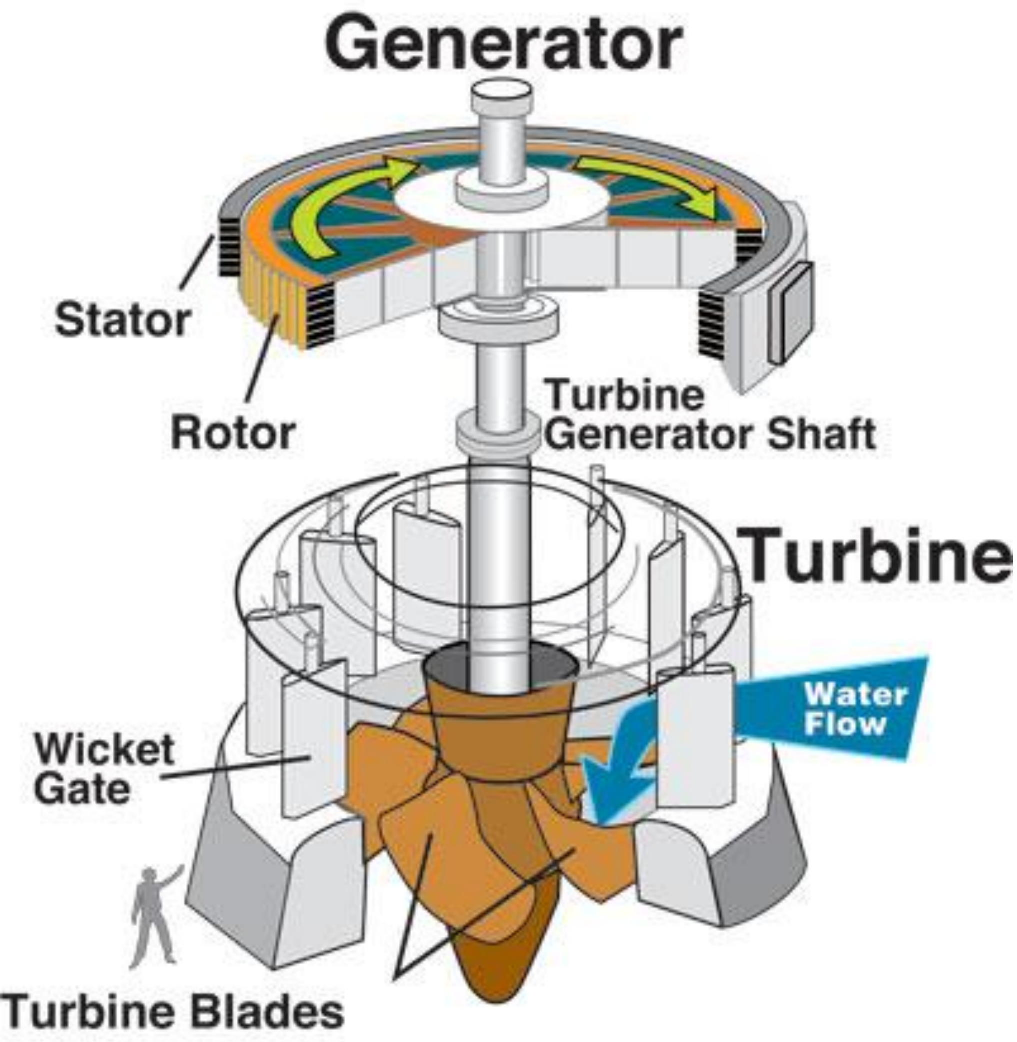
MQS applications:

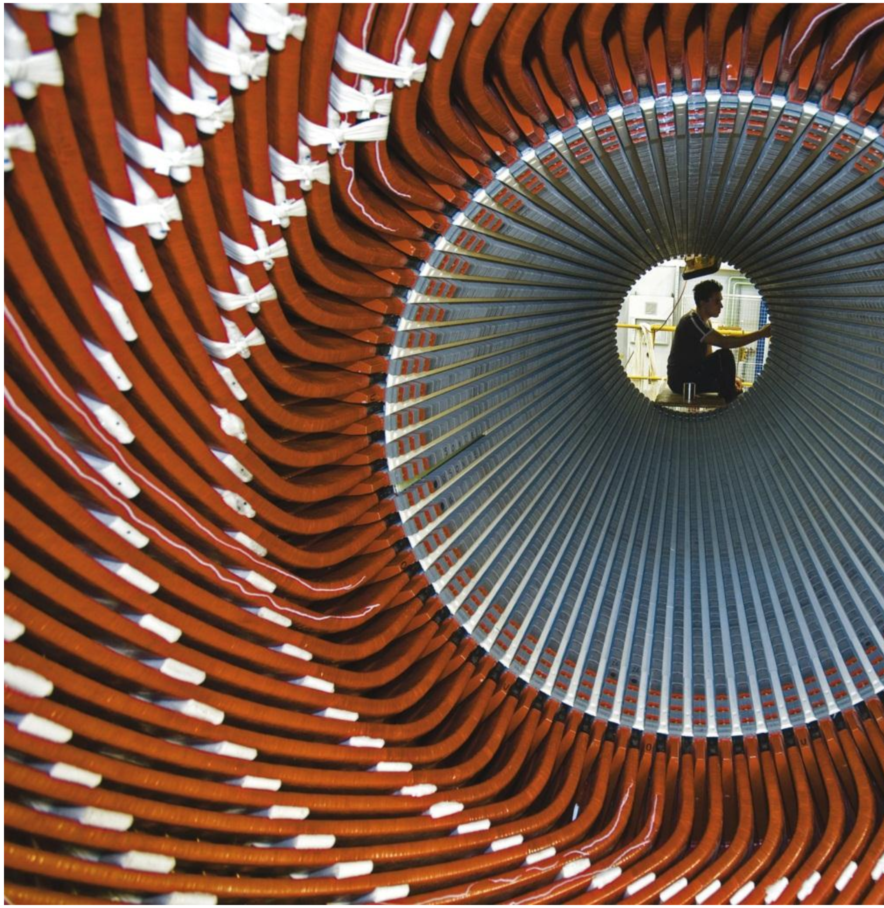
Counters, motors, trafos





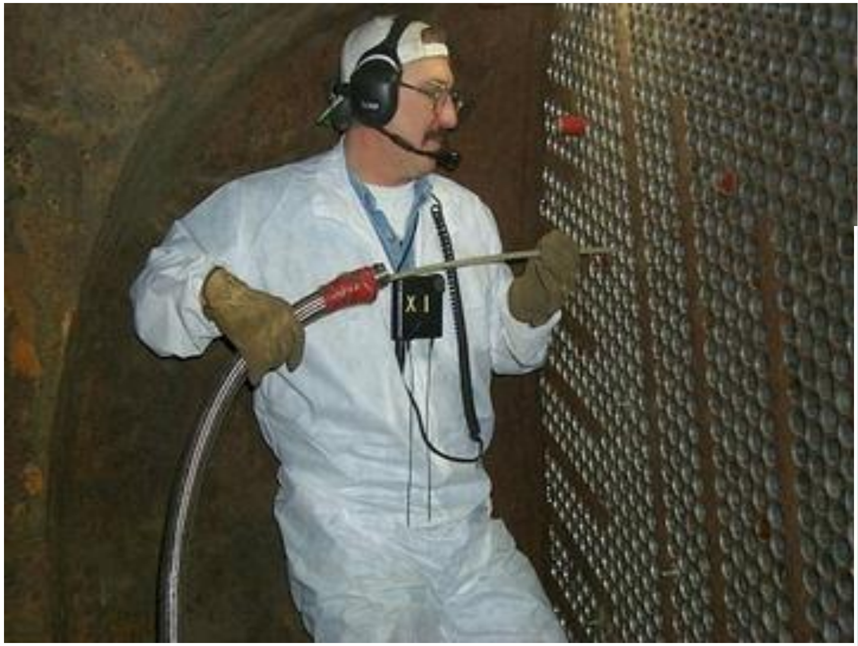
Hydro- and turbo-generators





Eddy current testing

- **Nuclear plants** **Air crafts**



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Not so easy questions, for curious people

1. How are first order MQS equations?
2. What type of potentials may be defined in MQS regime?
3. How are the second order equations for these potentials?
4. How are the boundary conditions for each potential to be unique?
5. How may be proven the uniqueness theorem?
6. What are MQS boundary conditions in semi-bounded domains?
7. How may be defined magnetic circuits in MQS regime?
8. What space may be used for trial and test functions in weak MQS formulation?
9. How are the integral equations of MQS field?
10. What about solution existence? What is a curl-curl operator?
11. What about nonlinear magnetic materials?
12. What about hysteresis?
13. What are the main novelties and difficulties of MQS regime, compared with static and steady state regimes?
14. Which of TEAM benchmarks are MQS problems?