Bazele Electrotehnicii

2. Legile electromgnetismului

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2. Legile electromagnetismului

Lege a unei stiinte = este o afirmatie fundamentala, care nu este demonstrata ci rezulta prin generalizarea unor observatiii si prin rationamente inductive incomplete.

- In teoriile stiintifice axiomatizate sistemul legilor trebuie sa indeplineasca urmatoarele conditii:
  - **Independenta** – orice lege nu este o consecinta logica a celorlante (nu poate fi dedusa din acestea)
  - **Consistenta** (noncontradictia) – nici o lege nu intra in contradictie logica cu celelalte sau cu o consecinta a lor
  - **Completitudinea** – sistemul legilor este suficient pentru a decide daca orice afirmatie corect formulata este adevarata sau falsa.
- Legile se exprima ca relatii matematice intre marimile primitive. Acest sistem de ecuatii trebuie sa conduca la probleme corect formulate matematic (care au solutie si aceasta este unica).
- Se presupune ca legile sunt complete si din punct de vedere fizic nu doar logico-matematic, adica orice fenomen este descris corect de consecintele legilor. In realitate, se condidera din domeniul teoriei doar acele fenomene ce sunt descrise corect.

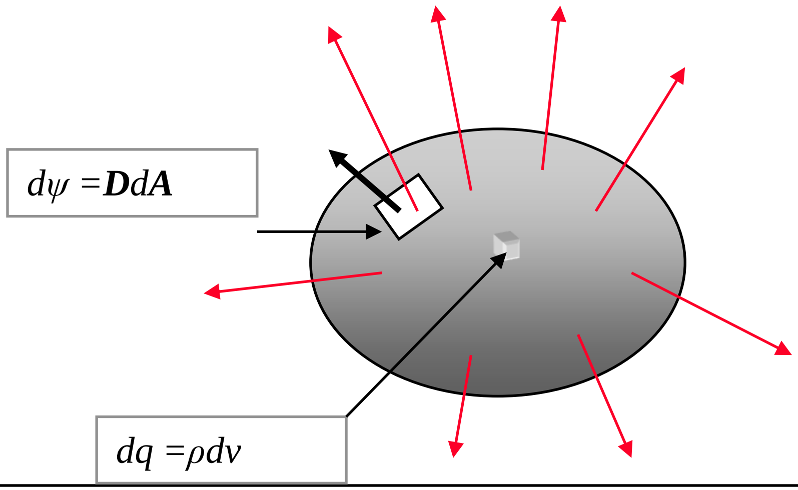
2.1. Legea fluxului electric (Gauss)

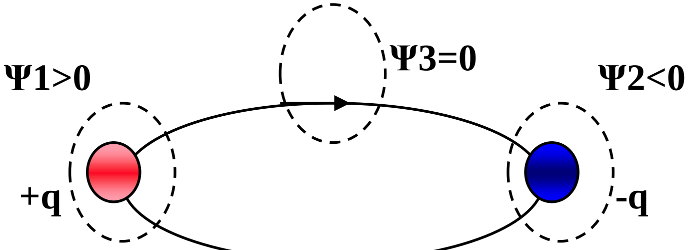
Enunt: Fluxul electric pe orice suprafata inchisa este egal cu sarcina electrica din interiorul suprafetei

Forma globala – integrala a legii:

$$\boxed{\psi_{\Sigma} = q_{D_{\Sigma}}} \Leftrightarrow \oint_{\Sigma} \mathbf{D}d\mathbf{A} = \int_{D_{\Sigma}} \rho dv$$

- **Semnificatie fizica**: orice corp electrizat produce camp electric
- **Consecinta calitativa** (forma si orientarea liniilor campului electric): Liniile campului inductiei electrice D sunt curbe deschise, ce pormesc de pe sarcinile pozitive si se opresc pe cele negative. Ele sunt neintrerupte in domeniile neutre.





Forma local a legii fluxului electric

- **Forma locala-diferentiala a legii**

$$div\mathbf{D} = \rho$$

- **Demonstratie** bazata pe teorema Gauss- Ostrogradski

$$\forall D_\Sigma : \oint_{\Sigma=\partial D_\Sigma} \mathbf{D} d\mathbf{A} = \int_{D_\Sigma} div\mathbf{D} dv = \int_{D_\Sigma} \rho dv \Rightarrow div\mathbf{D} = \rho$$

- **Semnificatia divergentei:**

$$div\mathbf{D} = \lim_{V_D \to 0} \oint_{\partial D} \mathbf{D} d\mathbf{A} / V_D$$

- productivitatea unui punct in linii de camp (nr de linii pe unitatea elemnatatara de volum) – pozitiva in izvor si negativa in sorb (daca este nula, linia este continua).

$$div\mathbf{D} = \nabla\mathbf{D} = (\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z})(\mathbf{i}D_x + \mathbf{j}D_y + \mathbf{k}D_z) = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

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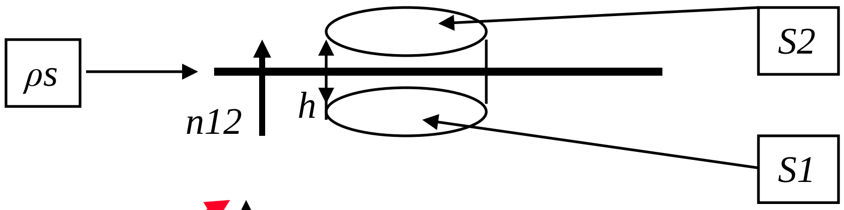
- **Forma pe suprafete de discontinuitate:**

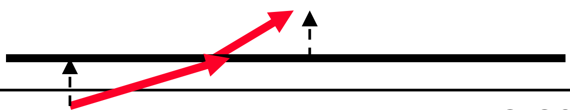
$$\oint_{\Sigma} \mathbf{D} d\mathbf{A} = \int_{S2} \mathbf{D} d\mathbf{A} + \int_{S1} \mathbf{D} d\mathbf{A} = \mathbf{n}_{12} \cdot (\mathbf{D}_2 - \mathbf{D}_1)_{ave} A =$$

$$\int_{D} \rho dv = \rho_v A h + \rho_s A \underset{h \to 0}{\Longrightarrow} \mathbf{n}_{12} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = \rho_s \Leftrightarrow div_s \mathbf{D} = \rho_s$$



- **Conservarea componentei normale a inductiei pe suprafete nelectrizate** (ρ_s=0):

$$\mathbf{n} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = 0 \Leftrightarrow D_{n1} = D_{n2}$$

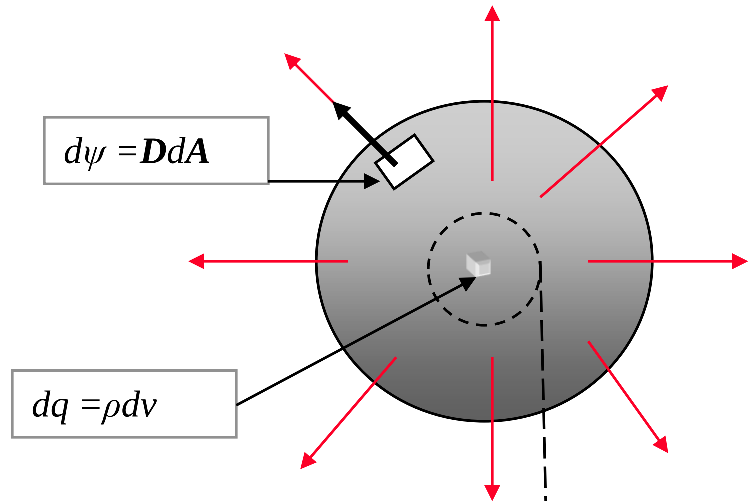


Campul electric produs de o sfera electrizata uniform

$$\psi_\Sigma = \oint_\Sigma \mathbf{D} d\mathbf{A} = \oint_\Sigma D dA = D \oint_\Sigma dA = D 4\pi R^2$$

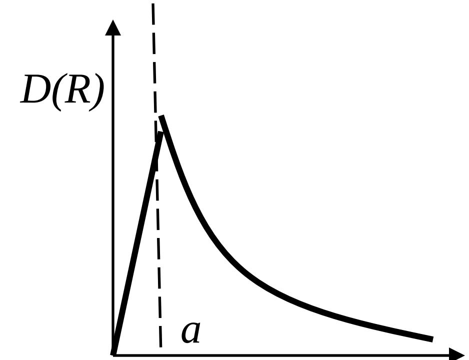
$$q_{D_\Sigma} = \int_{D_\Sigma} \rho dv = \rho \int_{D_\Sigma} dv = \rho \frac{4\pi R^3}{3}, \ for R < a$$

$$\psi_\Sigma = q_{D_\Sigma} \Rightarrow D_{\text{int}} 4\pi R^2 = \rho \frac{4\pi R^3}{3} \Rightarrow D_{\text{int}} = \rho \frac{R}{3}$$



$$q_{D_\Sigma} = \int_{D_\Sigma} \rho dv = \rho \int_\Omega dv = \rho \frac{4\pi a^3}{3} = q, \ for R > a$$

$$\psi_\Sigma = q_{D_\Sigma} \Rightarrow D_{ext} 4\pi R^2 = \rho \frac{4\pi a^3}{3} \Rightarrow D_{ext} = \rho \frac{a^3}{3R^2} = \frac{q}{4\pi R^2}; \mathbf{D} = \frac{q\mathbf{R}}{4\pi R^3}$$



Prin superpozitie : integrala coulombiana a inductiei produsa in vid de o distributie arbitrara de sarcina electrica :

$$\mathbf{D} = \int_\Omega \frac{\rho \mathbf{R} \mathrm{dv}}{4\pi R^3}$$

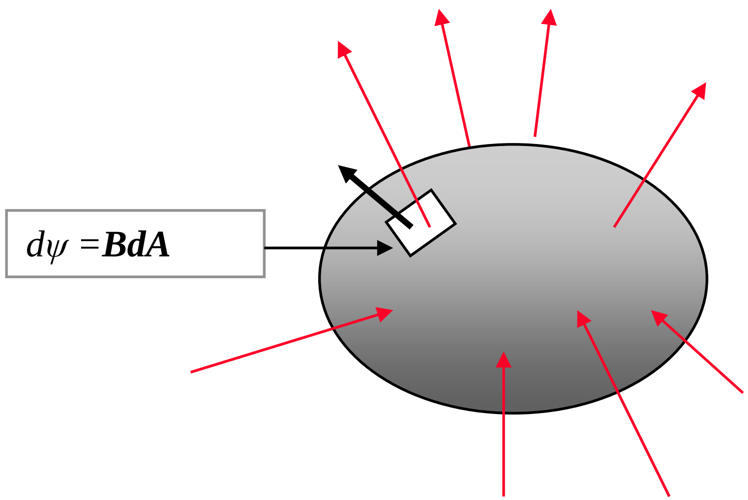
2.2. Legea fluxului magnetic

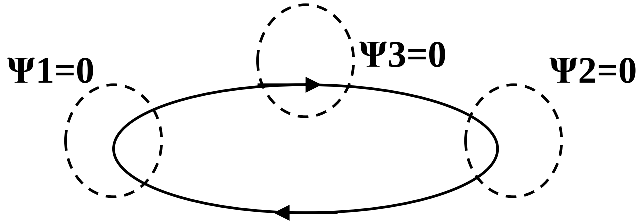
1. **Enunt:** Fluxul magnetic pe orice suprafata inchisa este nul
2. **Forma gobala (integrala) a legii:** $\varphi_\Sigma = 0 \Leftrightarrow \oint_\Sigma \mathbf{B} d\mathbf{A} = 0$
3. **Semnificatie fizica:**

nu exista “sarcini magnetice”

3. **Liniile campului magnetic:**

– Continui (fara punct de inceput sau sfarsit) – curbe inchise





Forma locala a legii fluxului magnetic

1. **Forma local (diferentiala) a legii:**

$$div\mathbf{B} = 0$$

2. **Demo** (bazata pe teorema Gauss Ostrogradski):

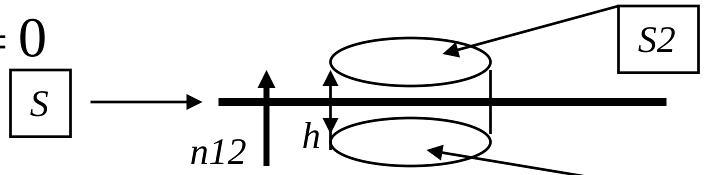
$$\forall D_\Sigma : \oint_{\Sigma=\partial D_\Sigma} \mathbf{B}d\mathbf{A} = \int_{D_\Sigma} div\mathbf{B}dv = 0 \Rightarrow div\mathbf{B} = 0$$

3. **Forma de ecuatie cu derivate partiale in coord. Cartezene**

$$div\mathbf{B} = \nabla\mathbf{B} = (\mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z})(\mathbf{i}B_x + \mathbf{j}B_y + \mathbf{k}B_z) = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

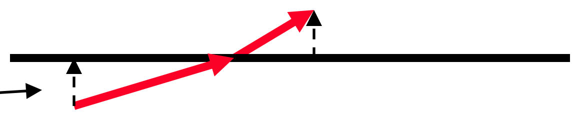
4. **Conservarea componentei normale a inductiei magnetice:**

$$\oint_\Sigma \mathbf{B}d\mathbf{A} = \int_{S2} \mathbf{B}d\mathbf{A} + \int_{S1} \mathbf{B}d\mathbf{A} = \mathbf{n}_{12} \cdot (\mathbf{B}_2 - \mathbf{B}_1)_{ave} A = 0$$



$$\underset{h\to 0}{\Longrightarrow} \mathbf{n}_{12} \cdot (\mathbf{B}_2 - \mathbf{B}_1) = 0$$

$$\mathbf{n} \cdot (\mathbf{B}_2 - \mathbf{B}_1) = 0 \Rightarrow B_{n1} = B_{n2}$$

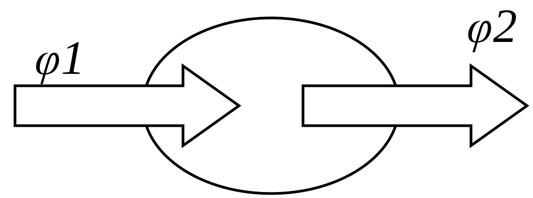


Consecinte integrale

1. Conservarea (continuitatea fluxului magnetic

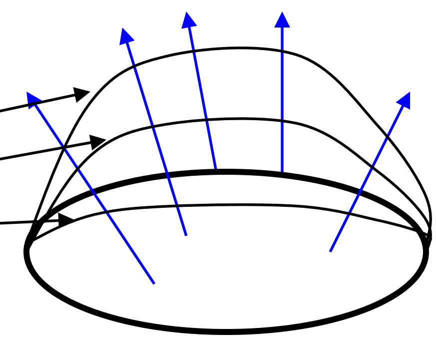
$$\varphi = \oint_{\Sigma = S1 \cup S2} \mathbf{B} d\mathbf{A} = \int_{S1} \mathbf{B} d\mathbf{A} + \int_{S2} \mathbf{B} d\mathbf{A} =$$

$$\int_{S1} \mathbf{B} d\mathbf{A}_1 - \int_{S2} \mathbf{B} d\mathbf{A}_2 = \varphi_1 - \varphi_2 = 0 \Longrightarrow \boxed{\varphi_1 = \varphi_2}$$



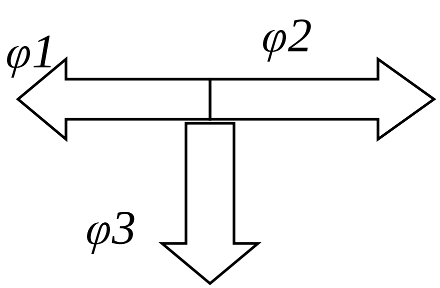
2. Invarianta fluxului magnetic fata de forma suprafetei (cu bordura fixa)

$$\varphi_1 = \varphi_2 = \varphi_2 \ldots$$



3. Relatia lui Kirchhoff pentru fluxurile din circuitele magnetice

$$\boxed{\sum_{k \in (n)} \varphi_k = 0}$$



Potentialul magnetic vector (optional)

1. **Definitia potentialului magnetic vector**

$$divB = 0 \Rightarrow \boxed{B = rotA}$$

$$divB = divrotA = \nabla \cdot (\nabla \times A) = 0$$

$$\varphi = \int_S BdS = \int_S rotAdS = \int_{\partial S} Adr \Rightarrow \boxed{\varphi = \int_{\partial S} Adr}$$

2. **Definitia rotorului:**

$$rotA = n \lim_{A_s \to 0} \oint_{\partial S} Adr / A_s$$

3. **Forma in coord. carteziene:**

$$rotA = \nabla \times A = (i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}) \times (iA_x + jA_y + kA_z) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

4. **Conditia de etalonare:**

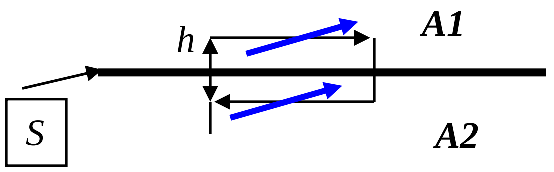
$$B = rotA = rot(A + A_0) \Rightarrow rotA_0 = 0 \Rightarrow A_0 = grad\lambda$$

$$divA = ?, \boxed{divA = \beta(=0)} \Rightarrow divA_0 = divgrad\lambda = \Delta\lambda = 0$$

5. **Continuitate potentialului vector (a componentei tangentiale)**

$$\varphi = B_{ave}lh = \int_{\partial S} Adr = (A_{t1} - A_{t2})l \to 0 \Rightarrow (A_{t1} - A_{t2}) = 0, \forall n_S \Rightarrow A_{t1} = A_{t2}$$

$$divA = 0 \Rightarrow A_{n1} = A_{n2} \Rightarrow \boxed{A_1 = A_2}$$



Recapitularea legilor de flux

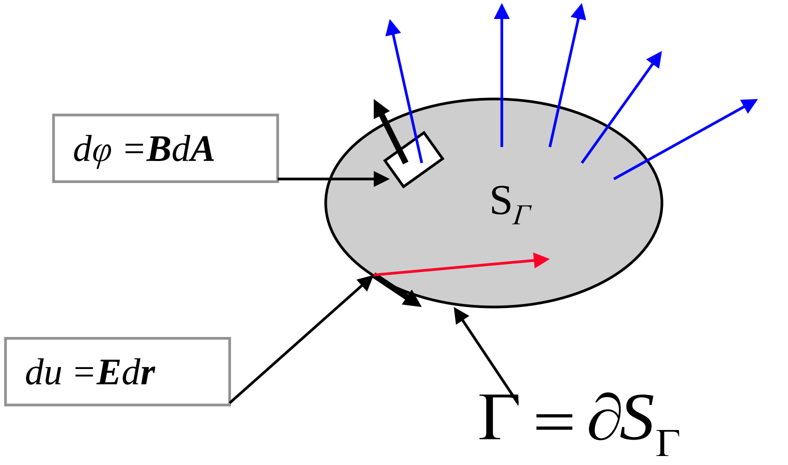
Camp:	Electric	Magnetic
Global	$\psi_\Sigma = q_{D_\Sigma}$	$\varphi_\Sigma = 0$
Integral	$\oint_\Sigma \mathbf{D}d\mathbf{A} = \int_{D_\Sigma} \rho dv$	$\oint_\Sigma \mathbf{B}d\mathbf{A} = 0$
Local diferential	$div\mathbf{D} = \rho$	$div\mathbf{B} = 0$
Pe supr de dics.	$div_s\mathbf{D} = \rho_s \Leftrightarrow$ $\mathbf{n}_{12}\cdot(\mathbf{D}_2 - \mathbf{D}_1) = \rho_s$	$div_s\mathbf{B} = 0 \Leftrightarrow$ $\mathbf{n}_{12}\cdot(\mathbf{B}_2 - \mathbf{B}_1) = 0$
Conserv.	$D_{n1} = D_{n2}$	$B_{n1} = B_{n2}$
Linii de camp	Deschise, orientate de la sarcinile pozitive la cele negative	Continui, inchise

2.2. Legea inductiei electromagnetice - Faraday

$$u_\Gamma = -\frac{d\varphi_{S\Gamma}}{dt} \Leftrightarrow \oint_\Gamma \mathbf{E}d\mathbf{r} = -\frac{d}{dt}\int_{S_\Gamma} \mathbf{B}d\mathbf{A}$$

1. **Enunt:** Tensiunea electrica pe orice curba inchisa este egala cu viteza de scadere a fluxului magnetic de pe orice suprafata ce se sprijina pe curba respectiva

2. **Forma globala (integrala) a legii:**

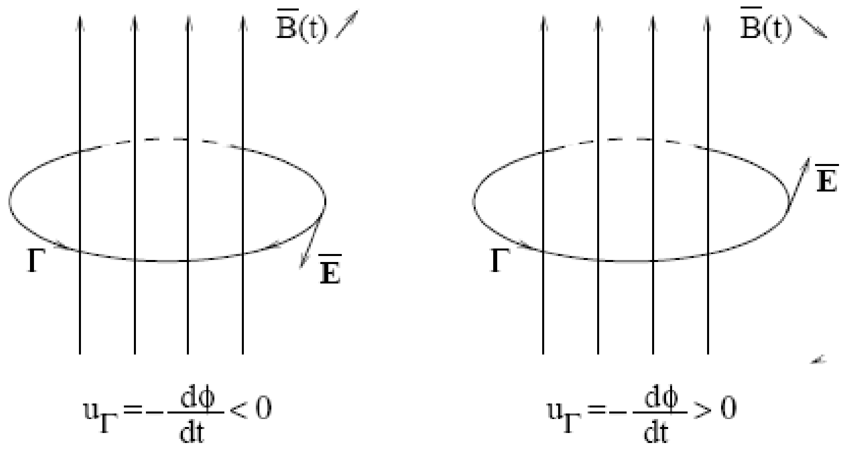


3. Semnificatie fizica: Variatia in timp a campului magnetic induce camp electric

4. Ipoteza Hertz: curba Γ si suprafata SΓ sunt antrenate de corpuri in miscarea lor. In consecinta iductia poate fi de transformare si/sau de miscare

5. Liniile campului electric indus:

- **Sunt curbe inchise**
- **Inconjoara campul magnetic inductor**
- **Sensul lor depinde de sensul liniilor campului magnetic inductor si de modul de variatie al acestuia**
- **Campul electric indus de un camp magnetic descrescator in timp are sensul dat de regula burghiului drept si este opus in cazul campu inductor crescator**



Forma local a legii inductiei in medii imobile

1. **Forma locala (diferentiala) a legii:**

$$rot\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \Leftrightarrow \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

2. **Proof (based on Stokes theorem):**

$$\oint_{\Gamma} \mathbf{E} d\mathbf{r} = \int_{S_\Gamma} curl \mathbf{E} d\mathbf{A} = -\frac{d}{dt} \int_{S_\Gamma} \mathbf{B} d\mathbf{A} = -\int_{S_\Gamma} \frac{\partial \mathbf{B}}{\partial t} d\mathbf{A}, \forall S_\Gamma \Uparrow$$

3. **Conservation of the tangential component of the electric field strength**

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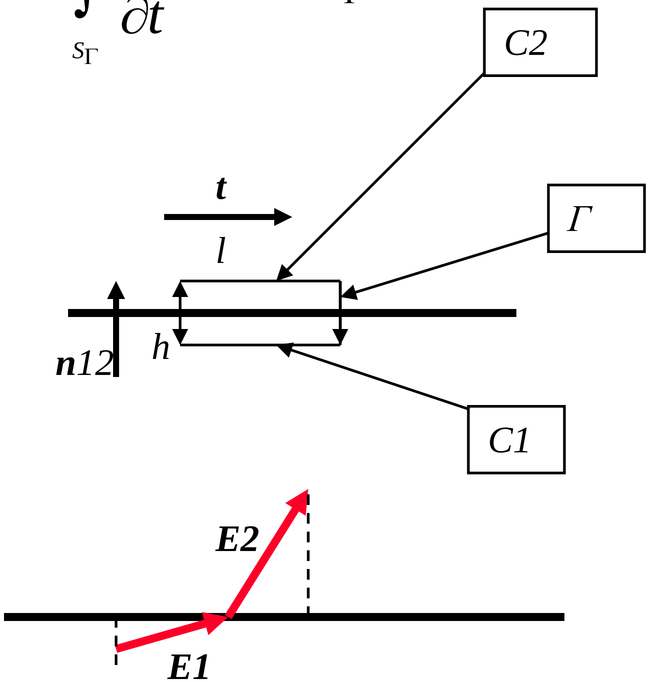
$$\oint_{\Gamma} \mathbf{E} d\mathbf{r} = \oint_{C1} \mathbf{E} d\mathbf{r} + \oint_{C2} \mathbf{E} d\mathbf{r} = \mathbf{t} \cdot (\mathbf{E}_2 - \mathbf{E}_1)_{ave} l,$$

$$\frac{d}{dt} \int_{S_\Gamma} \mathbf{B} d\mathbf{A} = \frac{dB_{nave}}{dt} lh \to 0, \text{when } h \to 0,$$

$$\Rightarrow \mathbf{n}_{12} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0, \Leftrightarrow \mathbf{E}_{t2} = \mathbf{E}_{t1}$$

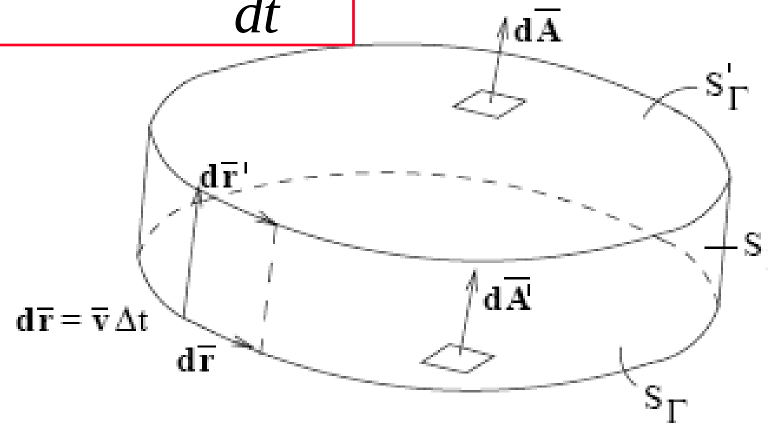


Formele local si integrala dezvoltata in medii mobile(optional)

1. **Forma locala (diferentiala):**

$$curl\,\mathbf{E} = -\frac{d_f\mathbf{B}}{dt}$$

2. **Demo (bazata pe teorema Stokes si derivata de flux:**



$$\frac{d}{dt}\int_{S_{\Gamma(t)}}\mathbf{B}(t)d\mathbf{A} = \frac{d}{dt}\int_{S_{\Gamma(t)}}\mathbf{B}d\mathbf{A} + \frac{d}{dt}\int_{S_\Gamma}\mathbf{B}(t)d\mathbf{A} \Rightarrow$$

$$\frac{d}{dt}\int_{S_{\Gamma(t)}}\mathbf{B}d\mathbf{A} = \lim_{\Delta t\to 0}\Big(\int_{S'_\Gamma}\mathbf{B}d\mathbf{A}' - \int_{S_\Gamma}\mathbf{B}d\mathbf{A}\Big)/\Delta t = -\lim_{\Delta t\to 0}\int_{Sl}\mathbf{B}d\mathbf{A}'/\Delta t = -\oint_{\Gamma}\mathbf{B}(d\mathbf{r}\times\mathbf{v}) = \int_{S_\Gamma} curl(\mathbf{B}\times\mathbf{v})d\mathbf{A}$$

$$\text{because } 0 = \oint_{\Sigma}\mathbf{B}d\mathbf{A} = \int_{S'_\Gamma}\mathbf{B}d\mathbf{A} - \int_{S_\Gamma}\mathbf{B}d\mathbf{A}' + \int_{Sl}\mathbf{B}d\mathbf{A} \text{ and on } Sl\; d\mathbf{A} = d\mathbf{r}\times\mathbf{s} = \Delta t(d\mathbf{r}\times\mathbf{v})$$

$$\frac{d}{dt}\int_{S_\Gamma}\mathbf{B}(t)d\mathbf{A} = \int_{S_\Gamma}\frac{\partial\mathbf{B}(t)}{\partial t}d\mathbf{A}, \Rightarrow \frac{d}{dt}\int_{S_{\Gamma(t)}}\mathbf{B}(t)d\mathbf{A} = \int_{S_{\Gamma(t)}}\frac{d_f\mathbf{B}(t)}{dt}d\mathbf{A}, \text{ where}$$

$$\frac{d_f\mathbf{B}(t)}{dt} =_{def} \frac{\partial\mathbf{B}}{\partial t} + curl(\mathbf{B}\times\mathbf{v})$$

$$curl\,\mathbf{E} = -\frac{d_f\mathbf{B}}{dt} \Leftrightarrow curl\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} - curl(\mathbf{B}\times\mathbf{v}) \Rightarrow$$

$$\oint_{\Gamma}(\mathbf{E}+\mathbf{B}\times\mathbf{v})d\mathbf{r} + \int_{S_\Gamma}\frac{\partial\mathbf{B}}{\partial t}d\mathbf{A} = 0$$

3. Forma integrala dezvoltata

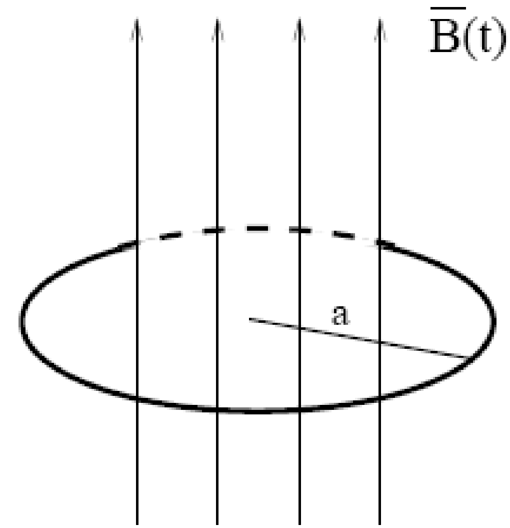
Aplicatii ale legii inductiei

1. Principiul transformatorului (inducta in medii imobile)

http://en.wikipedia.org/wiki/Transformer

$$\mathbf{B}(t) = \mathbf{k}B_0 \sin(\omega t),$$

$$u_\Gamma = -\frac{d\varphi_{S\Gamma}}{dt} = -AB_0\omega\cos(\omega t)$$



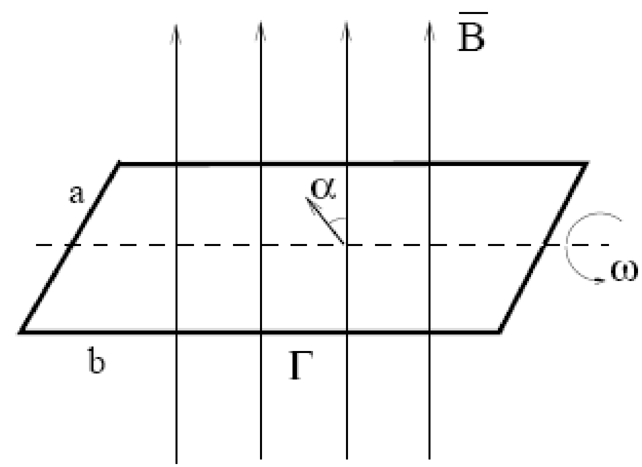
2. Principiul generatorului de tensiun alternativa (inductie de miscare in camp magnetic constant)

http://en.wikipedia.org/wiki/Alternator

$$\varphi_{S\Gamma} = \int_{S_\Gamma} \mathbf{B}\,d\mathbf{A} = BA\cos(\omega t),$$

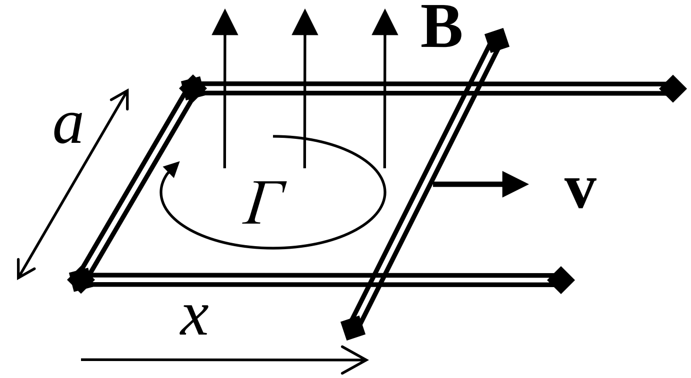
$$u_\Gamma = -\frac{d\varphi_{S\Gamma}}{dt} = BA\omega\sin(\omega t) \text{ or}$$

$$u_\Gamma = -\int_\Gamma (\mathbf{B}\times\mathbf{v})d\mathbf{r} = B\omega ab\sin(\omega t)$$



Aplicatii ale legii inductiei

3. T.e.m. indusa prin miscare liniara



$$e_\Gamma = -\frac{d\varphi_{S_\Gamma}}{dt} = -\frac{d}{dt}\int_{S_\Gamma} \mathbf{B} d\mathbf{A} = -Ba\frac{dx}{dt} = -Bav$$

$$e_\Gamma = \oint_\Gamma \mathbf{E} d\mathbf{r} = -\int_0^a Bv dr = -\int_{S_\Gamma} (\mathbf{B}\times\mathbf{v}) d\mathbf{r} \Rightarrow$$

$$\mathbf{E}_m = -\mathbf{B}\times\mathbf{v}$$

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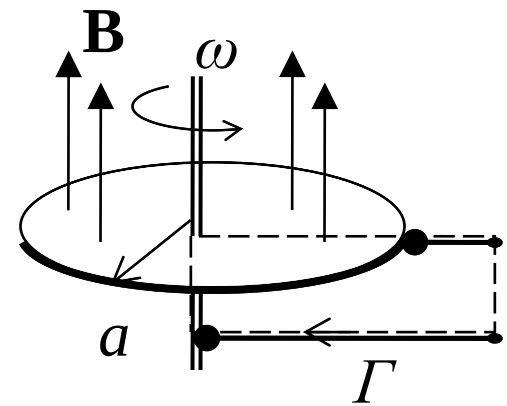
4. Principiul generatorului homopolar (de c.c.)

http://en.wikipedia.org/wiki/Homopolar_generator



$$e_\Gamma = -\oint_\Gamma (\mathbf{B}\times\mathbf{v}) d\mathbf{r} = \int_0^a B\omega r dr = B\omega a^2/2$$

Teorema potentialului electric stationar

In campuri stationare imobile:

$$u_\Gamma = 0 \Leftrightarrow \oint \mathbf{E} d\mathbf{r} = 0, \forall S_\Gamma$$

- **Forma globala**
- **Forma locala (diferentiala)**

$$curl\mathbf{E} = 0 \Rightarrow \mathbf{E} = -gradV \Leftrightarrow \mathbf{E} = -\nabla V$$

- **In coordonate carteziene:**

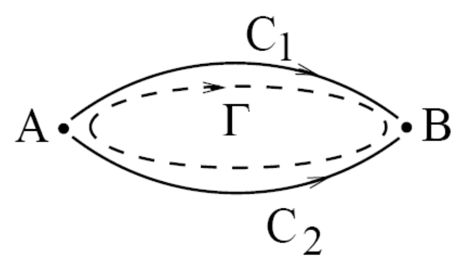
$$\mathbf{E} = -(\mathbf{i}\frac{\partial V}{\partial x} + \mathbf{j}\frac{\partial V}{\partial y} + \mathbf{k}\frac{\partial V}{\partial k})$$

- **Demo**:

$$curl\mathbf{E} = -curl(gradV) = -\nabla \times (\nabla V) = 0$$

- **Definitia gradientului** (grad indica directia, sensul si viteza spatiala de crestere a marimii scalare).

$$gradV = \lim_{W \to 0} \frac{1}{W} \int_{\partial\Omega} V d\mathbf{A}, \text{ unde } W = Vol(\Omega) = \int_{\Omega} dv$$



- **Independenta tensiunii electrice stationare de forma curbei**

$$u_\Gamma = \oint_\Gamma \mathbf{E} d\mathbf{r} = \int_{C1} \mathbf{E} d\mathbf{r} + \int_{C2} \mathbf{E} d\mathbf{r} = \int_{C1} \mathbf{E} d\mathbf{r}_1 - \int_{C2} \mathbf{E} d\mathbf{r}_2 = u_1 - u_2 = 0 \Rightarrow$$

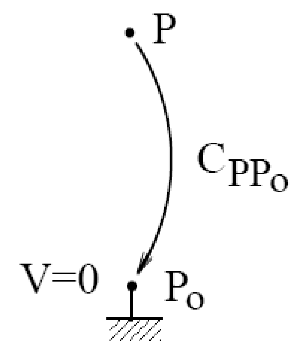
$$u_1 = u_2 \; \forall C_1, C_2 \text{ with } \partial C_1 = \partial C_2 = \{A, B\}$$

Definitia integrala a potentialului electric scalar

1. Unicitatea potentialului

$$\mathbf{E} = grad(V + C) = grad V$$

(este definit pana la o constanta aditiva care se fixeaza prin alegerea punctului de referinta (masa) in care V este conventional nul)



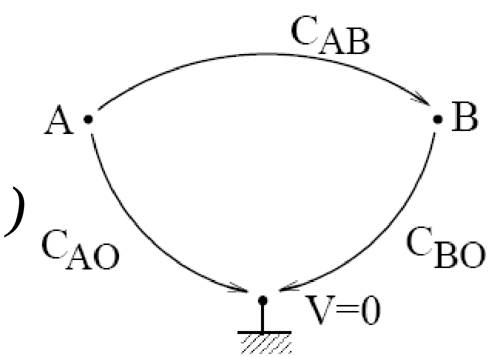
2. Definitia integrala a potentialului scalar (tensiunea pana la masa):

$$V(P) = \int_{C_{PO}} \mathbf{E} d\mathbf{r}$$

3. Demo:

$$\int_{C_{PO}} \mathbf{E} d\mathbf{r} = -\int_{C_{PO}} grad V d\mathbf{r} =$$

$$-\int_{C_{PO}} \left(\mathbf{i}\frac{\partial V}{\partial x} + \mathbf{j}\frac{\partial V}{\partial y} + \mathbf{k}\frac{\partial V}{\partial k}\right) d\mathbf{r} = -\int_{C_{PO}} dV = V(O) - V(P) = V(P)$$

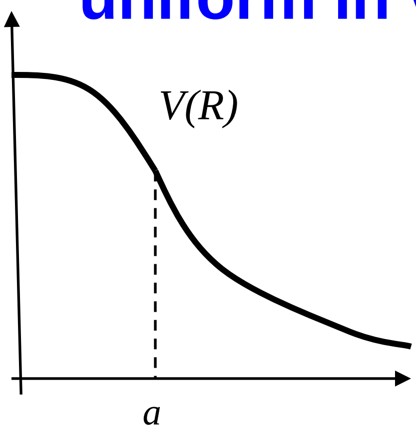


4. Tensiunea ca diferenta de potential:

$$u_{AB} = \int_{C_{AB}} \mathbf{E} d\mathbf{r} = \int_{C_{AO \cup OB}} \mathbf{E} d\mathbf{r} = \int_{C_{AO}} \mathbf{E} d\mathbf{r} - \int_{C_{OB}} \mathbf{E} d\mathbf{r} = V_A - V_B \Rightarrow u_{AB} = V_A - V_B$$

Aplicati ale teoremei potentialului stationar

- **Potentialul unei sfere electrizate uniform in vid**



Integrala coulombiana: potentialul unei distributii arbitrare de sarcini in vid:

$$dV = \frac{dq}{4\pi\varepsilon_0 R}; dq = \rho dv \Rightarrow$$

$$V(\mathbf{r}') = \int_\Omega \frac{\rho(\mathbf{r})dv}{4\pi\varepsilon_0 R}; R = |\mathbf{r}' - \mathbf{r}|$$

For $R > a, D = \frac{q}{4\pi R^2}$

$$D = \varepsilon_0 E \Rightarrow E = D/\varepsilon_0 = \frac{q}{4\pi\varepsilon_0 R^2}$$

$$V(R) = \int_{CPP_0} \mathbf{E}d\mathbf{r} = \frac{q}{4\pi\varepsilon_0}\int_{CPP_0}\frac{dr}{r^2} = -\frac{q}{4\pi\varepsilon_0 r}\Bigg|_R^{R_0} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{R_0}\right)$$

For $R_0 \to \infty$

$$V(R) = \frac{q}{4\pi\varepsilon_0 R}, \text{ where } R > a.$$

For $R < a, E = \frac{qR}{4\pi\varepsilon_0 a^3}$

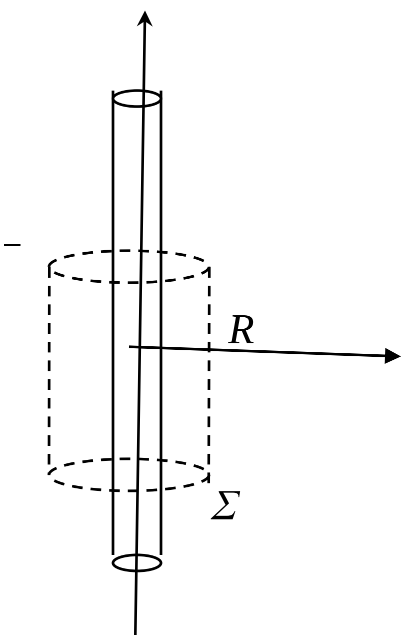
$$V(R) - V(a) = \int_{CPP_a}\mathbf{E}d\mathbf{r} = \frac{q}{4\pi\varepsilon_0 a^3}\int_{CPP_a} r dr = \frac{qr^2}{8\pi\varepsilon_0 a^3}\Bigg|_R^a = \frac{q(a^2 - R^2)}{8\pi\varepsilon_0 a^3}$$

$$V(R) = V(a) + \frac{q(a^2 - R^2)}{8\pi\varepsilon_0 a^3} = \frac{q}{4\pi\varepsilon_0 a} + \frac{q(a^2 - R^2)}{8\pi\varepsilon_0 a^3}$$

$$V(0) = \frac{3q}{8\pi\varepsilon_0 a}$$

$$V(R) = \begin{cases} \frac{q}{4\pi\varepsilon_0 a} + \frac{q(a^2 - R^2)}{8\pi\varepsilon_0 a^3} + C \text{ for } R < a; \\ \frac{q}{4\pi\varepsilon_0 R} + C, \text{ for } R > a. \end{cases}$$

Potentialul logaritmic – firul infinit electrizat



Pentru R > a; $\psi_\Sigma = q_{D_\Sigma} \Rightarrow 2\pi R l D = q \Rightarrow D = \frac{q}{2\pi R l} = \frac{\rho_l}{2\pi R}$

cu $\rho_l = \frac{q}{l}; D = \varepsilon_0 E \Rightarrow E = D/\varepsilon_0 = \frac{\rho_l}{2\pi\varepsilon_0 R}$

$$V(R) = \int_{CPP_0} \mathbf{E}\, d\mathbf{r} = \frac{\rho_l}{4\pi\varepsilon_0}\int_{CPP_0}\frac{dr}{r} = \frac{\rho_l \ln r}{2\pi\varepsilon_0}\Bigg|_{R}^{R_0} = \frac{\rho_l}{2\pi\varepsilon_0}(\ln R - \ln R_0)$$

$$V(R) = \frac{\rho_l}{2\pi\varepsilon_0}\ln\frac{R}{R_0} \qquad \text{pentru } R_0 = 1 \Rightarrow V(R) = \frac{\rho_l}{2\pi\varepsilon_0}\ln R,$$

Pentru $R < a$; $\psi_\Sigma = q_{D_\Sigma} \Rightarrow 2\pi R l D = \rho\pi R^2 l \Rightarrow D = \frac{\rho R}{2} = \frac{qR}{2\pi a^2 l} = \frac{R\rho_l}{2\pi a^2} \Rightarrow E = \frac{\rho_l R}{2\pi\varepsilon_0 a^2}$

$$V(R) - V(a) = \int_{CPP_a} \mathbf{E}\, d\mathbf{r} = \frac{\rho_l}{2\pi\varepsilon_0 a^2}\int_{CPP_a} r\,dr = \frac{\rho_l r^2}{4\pi\varepsilon_0 a^2}\Bigg|_{R}^{a} = \frac{\rho_l (a^2 - R^2)}{2\pi\varepsilon_0 a^2}$$

$$V(R) = V(a) + \frac{\rho_l (a^2 - R^2)}{2\pi\varepsilon_0 a^2} = \frac{\rho_l (a^2 - R^2)}{2\pi\varepsilon_0 a^2} + \frac{\rho_l}{2\pi\varepsilon_0}\ln\frac{a}{R_0}$$

$$V(0) = \frac{\rho_l}{2\pi\varepsilon_0}\left(1 + \ln\frac{a}{R_0}\right)$$

$$V(R) = \begin{cases} \frac{\rho_l (a^2 - R^2)}{2\pi\varepsilon_0 a^2} + \frac{\rho_l}{2\pi\varepsilon_0}\ln\frac{a}{R_0} + C; \text{pt } R < a; \\ \frac{\rho_l}{2\pi\varepsilon_0}\ln R + C; \text{pt } R > a. \end{cases}$$

Recapitularea legii inductiei electromagnetice

Miscare	Nu	Da
Global	$u_\Gamma = -\frac{d\varphi_{S\Gamma}}{dt}$	$u_\Gamma = -\frac{d\varphi_{S\Gamma}}{dt}$
Integral	$\oint_\Gamma \mathbf{E}d\mathbf{r} = -\frac{d}{dt}\int_{S_\Gamma} \mathbf{B}d\mathbf{A}$	$\oint_\Gamma \mathbf{E}d\mathbf{r} = -\frac{d}{dt}\int_{S_\Gamma} \mathbf{B}d\mathbf{A}$
Integral dezvoltata	$\oint_\Gamma \mathbf{E}d\mathbf{r} = -\int_{S_\Gamma} \frac{\partial \mathbf{B}}{\partial t} d\mathbf{A}$	$\oint_\Gamma \mathbf{E}d\mathbf{r} = -\int_{S_\Gamma} \frac{\partial \mathbf{B}}{\partial t} d\mathbf{A} - \oint_\Gamma (\mathbf{B}\times\mathbf{v})d\mathbf{r}$
Pe supraf.	$\mathbf{n}_{12}\times(\mathbf{E}_2 - \mathbf{E}_1) = 0$	$\mathbf{n}_{12}\times[(\mathbf{E}+\mathbf{B}\times\mathbf{v})_2 - (\mathbf{E}+\mathbf{B}\times\mathbf{v})_2] = 0$
Conserv.	$\mathbf{E}_{t1} = \mathbf{E}_{t2}$	-
Liniile campului indus	Inconjoara campul magnetic inductor	-

2.4.Legea circuitului magnetic - Ampere-Maxwell

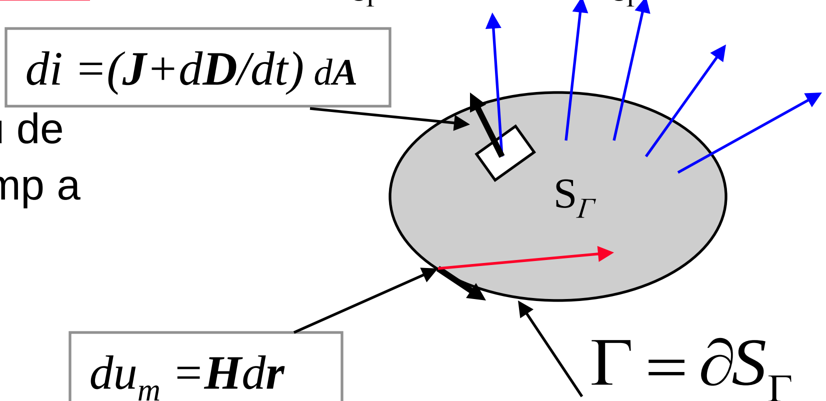
1. Enunt: Tensiunea magnetica pe orice curba inchisa este egala cu suma dintre curentul electric printr-o suprafata arbitrara care se sprijina pe acea curba si viteza de variate a fluxului electric de pe acea suprafata.

2. Forma globala (integrala) a legii:

$$u_{m\Gamma} = i_{S\Gamma} + \frac{d\psi_{S\Gamma}}{dt} \Leftrightarrow \oint_{\Gamma} \mathbf{H} d\mathbf{r} = \int_{S_\Gamma} \mathbf{J} d\mathbf{A} + \frac{d}{dt}\int_{S_\Gamma} \mathbf{D} d\mathbf{A}$$

3. Semnificatie fizica:

Orice curent electric (de conductie, deplasare sau de convectie produce camp magnetic). Variatia in timp a fluxului electric genereaza camp magnetic.

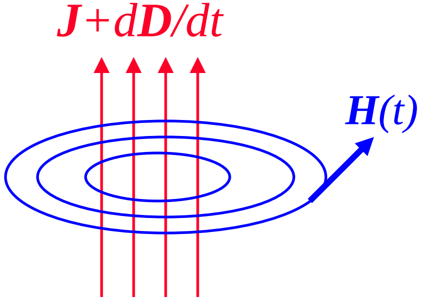


Viteza de variatie a fost numita de Maxwell curenat de deplasare, si are densitatea:

$$\mathbf{J}_d =_{def} \frac{\partial \mathbf{D}}{\partial t}$$

. **4. Liniile campului magnetic:**

- Sunt curbe inchise
- Inconjoara liniile de curent de conductie sau deplasare
- Sensul lor este dat de regula burgiului drept care avanseaza in sensul curentului



Forma locala a legii circuitului magnetic in medii imobile

1. Forma locala (diferentiala) in mediii imobile

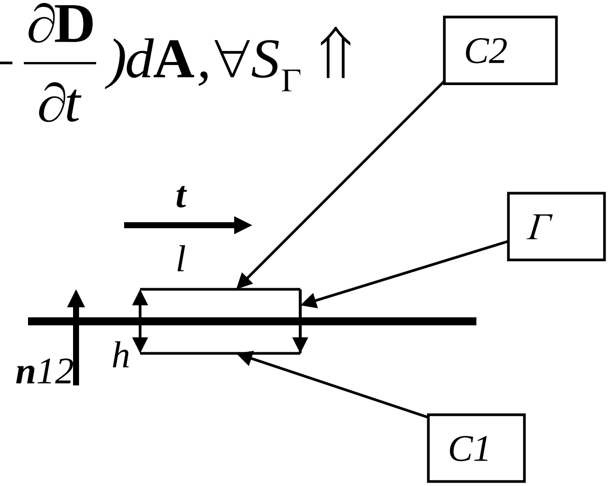
$$rot\mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \Leftrightarrow \nabla \times \mathbf{E} = \mathbf{J}_t$$

$\frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_d$, $\mathbf{J}_t = \mathbf{J} + \mathbf{J}_d$

Rotorul unui camp vectorial indica vartejurile acestui camp (axa si sensul de rotatie a liniilor de camp)

2. Demo (bazata oe teorema lui Stokes):

$$\oint_{\Gamma} \mathbf{H} d\mathbf{r} = \int_{S_\Gamma} curl\mathbf{H} d\mathbf{A} = \int_{S_\Gamma} \mathbf{J} d\mathbf{A} + \frac{d}{dt}\int_{S_\Gamma} \mathbf{D} d\mathbf{A} = \int_{S_\Gamma} (\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}) d\mathbf{A}, \forall S_\Gamma \Uparrow$$

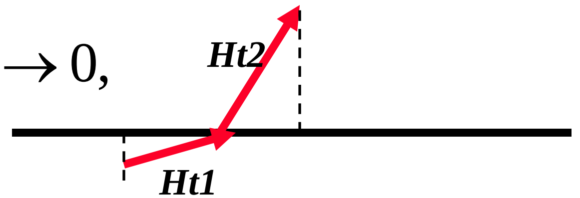


3. Conservarea componentei tangentiale a intensitatii campului magnetic:

$$\oint_{\Gamma} \mathbf{H} d\mathbf{r} = \oint_{C1} \mathbf{H} d\mathbf{r} + \oint_{C2} \mathbf{H} d\mathbf{r} = \mathbf{t} \cdot (\mathbf{H}_2 - \mathbf{H}_1)_{ave} l,$$

$$\int_{S_\Gamma} \mathbf{J} d\mathbf{A} + \frac{d}{dt}\int_{S_\Gamma} \mathbf{D} d\mathbf{A} = (J_{nave} + \frac{dD_{nave}}{dt}) lh \to J_s l, \text{cand } h \to 0,$$

$$\Rightarrow \mathbf{n}_{12} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{J}_s \Rightarrow \boxed{\mathbf{H}_{t2} = \mathbf{H}_{t1}}, \text{daca } J_s = 0.$$



Forma locala si cea integarla dezvoltata in medii mobile (optional)

1. **Forma locala (diferentiala):**

$$rot\,\mathbf{H} = \mathbf{J} + \frac{d_f\mathbf{D}}{dt} \Leftrightarrow$$

$$rot\,\mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} + \rho\mathbf{v} + rot(\mathbf{D}\times\mathbf{v})$$

2. **Demo,** bazata pe teorema lui Stokes si derivata de flux:

$$\frac{d}{dt}\int_{S_{\Gamma}(t)}\mathbf{D}(t)d\mathbf{A} = \frac{d}{dt}\int_{S_{\Gamma}(t)}\mathbf{D}d\mathbf{A} + \frac{d}{dt}\int_{S_{\Gamma}}\mathbf{D}(t)d\mathbf{A} \Rightarrow \frac{d}{dt}\int_{S_{\Gamma}(t)}\mathbf{D}d\mathbf{A} = \lim_{\Delta t\to 0}\left(\int_{S'_{\Gamma}}\mathbf{D}d\mathbf{A}' - \int_{S_{\Gamma}}\mathbf{D}d\mathbf{A}\right)/\Delta t =$$

$$-\int_{S_{\Gamma}}(\mathbf{v}\,div\mathbf{D} + curl(\mathbf{D}\times\mathbf{v}))d\mathbf{A},\ \frac{d}{dt}\int_{S_{\Gamma}}\mathbf{D}(t)d\mathbf{A} = \int_{S_{\Gamma}}\frac{\partial\mathbf{D}(t)}{\partial t}d\mathbf{A}, \Rightarrow \frac{d}{dt}\int_{S_{\Gamma}(t)}\mathbf{D}(t)d\mathbf{A} = \int_{S_{\Gamma}(t)}\frac{d_f\mathbf{D}(t)}{dt}d\mathbf{A},$$

$$\text{unde } \frac{d_f\mathbf{D}(t)}{dt} =_{def} \frac{\partial\mathbf{D}}{\partial t} + \mathbf{v}\,div\mathbf{D} + rot(\mathbf{D}\times\mathbf{v}),\ rot\,\mathbf{H} = \mathbf{J} + \frac{d_f\mathbf{D}}{dt};\ div\mathbf{D} = \rho \Rightarrow$$

$$rot\mathbf{H} = \mathbf{J} + \frac{\partial\mathbf{D}}{\partial t} + \rho\mathbf{v} + rot(\mathbf{D}\times\mathbf{v}) \Rightarrow \oint_{\Gamma}(-\mathbf{H} + \mathbf{D}\times\mathbf{v})d\mathbf{r} + \int_{S_{\Gamma}}\left(\rho\mathbf{v} + \frac{\partial\mathbf{D}}{\partial t}\right)d\mathbf{A} = 0$$

3. Forma integrala dezvoltata

Densitatilor curentilor de: **Coductie, Deplasare, Convectie si Rontgen teoretic.**

Doar Electrodinamica relativista da expresia corecta a curentului Rontgen!

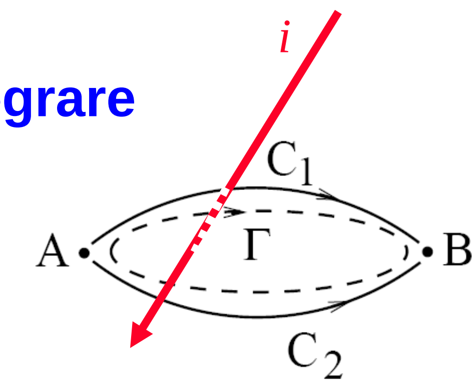
Teorema lui Ampere. Cazul stationar

In cazul campurilor stationare din mediile imobile:

- **Forma globala:** $u_{m\Gamma} = i_{S\Gamma} \Leftrightarrow \oint_{\Gamma} \mathbf{H}d\mathbf{r} = \int_{S_\Gamma} \mathbf{J}d\mathbf{A}, \forall S_\Gamma$
- **Forma local (diferentiala):** $rot\mathbf{H} = \mathbf{J} \Leftrightarrow \nabla \times \mathbf{H} = \mathbf{J}$
- **In coord. carteziene:**

$$curl\mathbf{H} = \nabla \times \mathbf{H} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \mathbf{J}$$

- **Demo:** $rot\mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}$
- **Dependenta tensiunii magnetice de calea de integrare**



$$u_{m\Gamma} = \oint_{\Gamma} \mathbf{H}d\mathbf{r} = \int_{C1} \mathbf{H}d\mathbf{r} + \int_{C2} \mathbf{H}d\mathbf{r} = \int_{C1} \mathbf{H}d\mathbf{r}_1 - \int_{C2} \mathbf{H}d\mathbf{r}_2 = u_1 - u_2 = i_{S\Gamma} \Rightarrow$$

$u_1 = u_2 \; \forall C_1, C_2$ cu $\partial C_1 = \partial C_2 = \{A, B\}$ numai daca $\mathbf{J} = 0$.

In acest caz poate fi definit un potential magnetic scalar: $\mathbf{H} = -gradV_m$

Aplicatii ale legii circuitului magnetic

Campul magnetic produs de un conductor cilindric parcurs longitudinal de un curent distribuit uniform

$$u_{m\Gamma} = \oint_{\Gamma} \mathbf{H} d\mathbf{r} = H \int_{\Gamma} dr = H 2\pi r,$$

$$i_{S\Gamma} = \int_{S_\Gamma} \mathbf{J} d\mathbf{A} = J \int_{S_\Gamma} dA = J\pi r^2, \text{ pt. } r < a$$

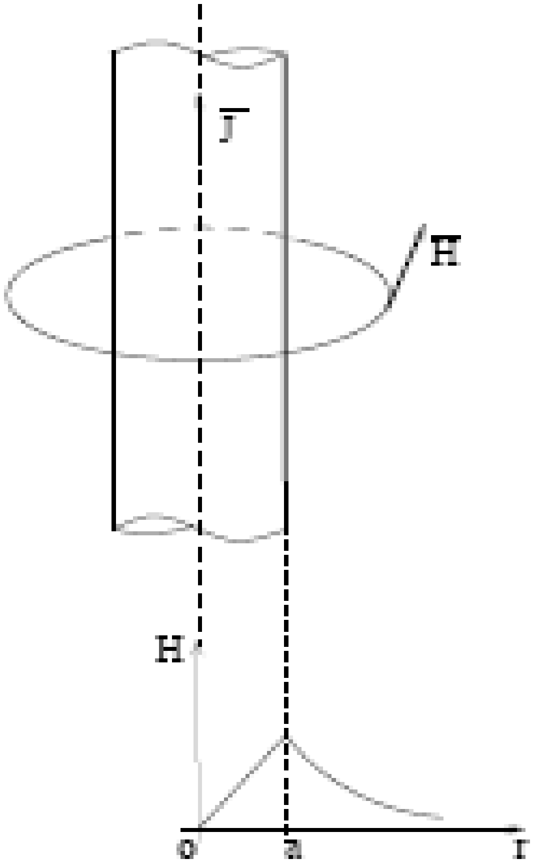
$$u_{m\Gamma} = i_{S\Gamma} \Rightarrow H = Jr/2$$

$$\text{For } r > a, i_{S\Gamma} = \int_{S_\Gamma} \mathbf{J} d\mathbf{A} = \int_{S_{\Gamma a}} J dA = J\pi a^2 = I,$$

–

$$H = \frac{Ja^2}{2r} = \frac{I}{2\pi r}$$

$$H(r) = \begin{cases} \frac{Jr}{2} = \frac{Ir}{2\pi a^2}, \text{pt.} r < a \\ \frac{Ja^2}{2r} = \frac{I}{2\pi r}, \text{pt.} r > a \end{cases}$$



Un camp electric uniform cu inductia variabila in timp $D(t)$ **produces produce acelas camp magnetic, daca** $J = dD/dt$**.**

Recapitularea legii circuitului magnetic

Miscare	Nu	Da
Global	$u_{m\Gamma} = i_{S\Gamma} + \frac{d\psi_{S\Gamma}}{dt}$	$u_{m\Gamma} = i_{S\Gamma} + \frac{d\psi_{S\Gamma}}{dt}$
Integral	$\oint_{\Gamma} \mathbf{H} d\mathbf{r} = \int_{S_\Gamma} \mathbf{J} d\mathbf{A} + \frac{d}{dt} \int_{S_\Gamma} \mathbf{D} d\mathbf{A}$	$\oint_{\Gamma} \mathbf{H} d\mathbf{r} = \int_{S_\Gamma} \mathbf{J} d\mathbf{A} + \frac{d}{dt} \int_{S_\Gamma} \mathbf{D} d\mathbf{A}$
Integral dezvoltat	$\oint_{\Gamma} \mathbf{H} d\mathbf{r} = \int_{S_\Gamma} \mathbf{J} d\mathbf{A} + \int_{S_\Gamma} \frac{\partial \mathbf{D}}{\partial t} d\mathbf{A}$	$\oint_{\Gamma} \mathbf{H} d\mathbf{r} = \int_{S_\Gamma} (\mathbf{J} + \rho \mathbf{v} + \frac{\partial \mathbf{D}}{\partial t}) d\mathbf{A} + \oint_{\Gamma} (\mathbf{D} \times \mathbf{v}) d\mathbf{r}$
Pe suprafete	$\mathbf{n}_{12} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{J}_s$	$\mathbf{n}_{12} \times [(\mathbf{H} + \mathbf{D} \times \mathbf{v})_2 - (\mathbf{H} + \mathbf{D} \times \mathbf{v})_2] = 0$
Conserv.	$\mathbf{H}_{t1} = \mathbf{H}_{t2}$	-
Linii de camp		-

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Legi generale si legi de material ale electromagetismului

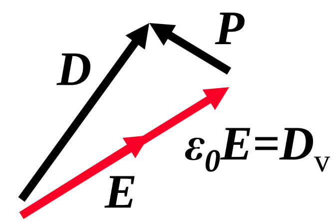
- Cele patru legi prezentate descriu fenomenele fundamentale ale e-mg si au caracter general numindu-se din acest motiv **legi generale**.
- Descriu cantitativ relatiile de cauzalitate intre sursele de camp si efectul lor (conform semnificatiileor fizice, campul electric are doua cauze iar campul magnetic alte doua cauze).
- Formele lor locale alcatuiec un sistem de 4 ecuatii PDE ce leaga intre ele marimile el-mg primitive (locale) ale campului si corpurilor.
- Pe interfetele intre corpuri se conserva componentele tangentiale ale intensitatilor si componentele normale ale inductiilor electrice si magnetice. In consecinta, tensiunile si fluxurile din corpuri nu se modifica daca in jurul curbeleor si suprafetelor de definitie se practica fante vide minuscule.
- Campul electric stationar nu rezula univoc din aceste legi, deoarece ele descriu divergenta lui D si rotorul campului E. Conform teoremei fundamentale a campurilor de vectori pentru a determina un camp trebuie date si divergenta si rotorul acestuia. Conditie este indeplinita doar daca este cunoscuta o relatie intre suplimentara E si D. Este deci necesara cel putin inca o lege, care sa descrie aceasta dependenta, dar si una pentru legatura B-H din camp magnetic si alta intre J si E.
- In teoria macroscopica aceaste dependente se stabilesc pe cale empirica (spre deosebire de teoriile microscopice, in care aceste relatii se deduc din structura intima a substantei). Forma concreta a relatiei deinzand de tipul substante, aceste relatii se numesc **legi de material**

2.5. Legea legaturii dintre D si E (a polarizatiei)

1. Enunt: Inductia electrica dintr-un punct din spatiu depinde de intensitatea campului elctric din acel punct. Forma concreta a dependentei este functie de substanta in care se afla punctul.

2. Forma generala, locala a legii: $\mathbf{D} = \mathbf{f}(\mathbf{E})$ $\quad \mathbf{f} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$

- caracteristica dielectrica



3. Forme locale particulare

- in vid: $\mathbf{D} = \varepsilon_0 \mathbf{E},\quad \varepsilon_0 \cong \dfrac{1}{4\pi 9 \cdot 10^9} F/m$
- in dielectrici liniari izotropi: $\mathbf{D} = \varepsilon \mathbf{E},\quad \varepsilon_r = \varepsilon / \varepsilon_0 \Rightarrow \varepsilon = \varepsilon_r \varepsilon_0$
- in in dielectrici liniari anizotropi: $\mathbf{D} = \overline{\overline{\varepsilon}} \mathbf{E},\quad \overline{\overline{\varepsilon}} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}, \varepsilon_{ij} = \varepsilon_{ji}, \mathbf{ED} = \mathbf{E}\overline{\overline{\varepsilon}}\mathbf{E} > 0$
- in corpuri polarizate permanent (modelul afin-aproximare de ordin 1): $\mathbf{D} = \overline{\overline{\varepsilon}} \mathbf{E} + \mathbf{P}_p$ primii doi termeni din seria Taylor
- in general (in dielectrici neliniari): $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \Rightarrow \mathbf{P} = \mathbf{D} - \varepsilon_0 \mathbf{E} = \mathbf{f}(\mathbf{E}) - \varepsilon_0 \mathbf{E} = \mathbf{P}_t(\mathbf{E}) + \mathbf{P}_p$

$\mathbf{P}_p =_{def} \mathbf{f}(0)$

- In dielectrici liniari: $\mathbf{P}_p = 0, \mathbf{P} = \mathbf{P}_t(\mathbf{E}) = \varepsilon_0 \chi \mathbf{E}, \mathbf{D} = \varepsilon_0 (1 + \chi) \mathbf{E} \Rightarrow \varepsilon_r = 1 + \chi$

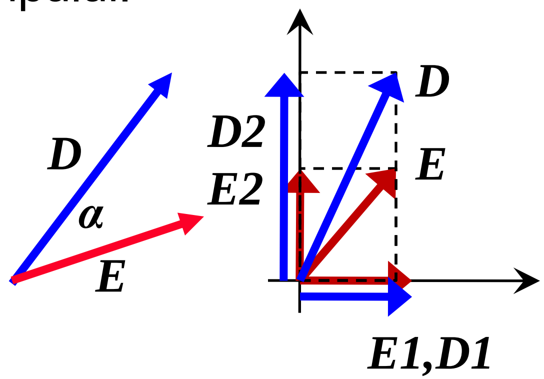
3. Semnificatii fizice: Polarizatia $\mathbf{P} = \mathbf{D} - \varepsilon_0 \mathbf{E} = \mathbf{f}(\mathbf{E}) - \varepsilon_0 \mathbf{E} = \mathbf{P}_t(\mathbf{E}) + \mathbf{P}_p [C/m^2]$ marime derivata: descrie polarizarea permanenta (din electreti) - sursa de camp electric si polarizatrea temporara – prin care dielectricii perturba campul in care se afla

Dielectrici anizotropi- directii si valori principale

1. In acest caz inductia electrica nu are aceeasi directie cu intensitatea campului. Unghiul dintre cei doi vectori depinde de orientarea campului.

2. Tensorul permitivitatii:

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \Rightarrow \begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \Leftrightarrow \mathbf{D} = \bar{\bar{\varepsilon}}\mathbf{E}$$



3. La schimbarea sistemului de referinta componentle vectorilor se modifica astfel:

$$\mathbf{D}' = T\mathbf{D}, \quad \mathbf{E}' = T\mathbf{E} \Rightarrow T^{-1}\mathbf{D}' = \bar{\bar{\varepsilon}}T^{-1}\mathbf{E}' \Rightarrow \mathbf{D}' = T\bar{\bar{\varepsilon}}T^{-1}\mathbf{E}' == \bar{\bar{\varepsilon}}'\mathbf{E}' \Rightarrow \bar{\bar{\varepsilon}}' = T\bar{\bar{\varepsilon}}T^{-1}$$

Tensorul sufera o transformare de similitudine

4. D si E sunt coliniari pentru directiile proprii (principale) ale tensorului, care se diagonalizeaza in sistemul de referinta cu axele in directiile principale:

$$\alpha = 0 \Rightarrow \mathbf{D} = \lambda\mathbf{E} \Rightarrow \bar{\bar{\varepsilon}}\mathbf{E} = \lambda\mathbf{E} \Rightarrow (\bar{\bar{\varepsilon}} - \lambda\bar{\bar{I}})\mathbf{E} = 0 \Rightarrow \det(\bar{\bar{\varepsilon}} - \lambda\bar{\bar{I}}) = 0 \Rightarrow \lambda_1 = \varepsilon_1; \lambda_2 = \varepsilon_2; \lambda_3 = \varepsilon_3$$

$$\boxed{\bar{\bar{\varepsilon}}' = diag(\varepsilon_1, \varepsilon_2, \varepsilon_3);} \quad \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon \Rightarrow \bar{\bar{\varepsilon}}' = \varepsilon\bar{\bar{I}} \Rightarrow \mathbf{D}' = \varepsilon\mathbf{E}'$$

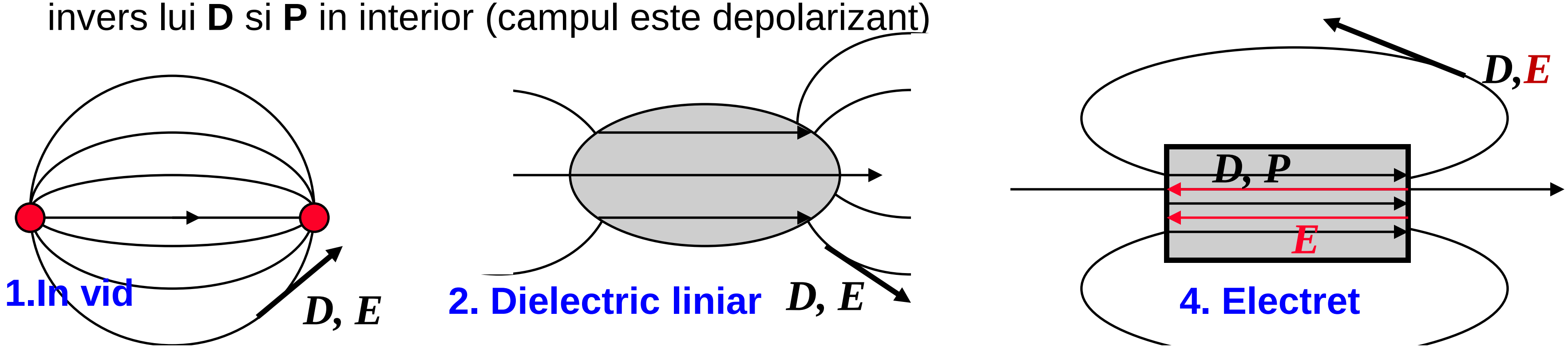
Campul electric in corpuri

1. In vid D si E au linii comune (este suficient un singur camp vectorial E sau D pentru descrie campul electric iar P=0). Inductia este proportionala cu intensitatea printr-o ct. universala

2. Datorita polarizarii lor temporare, dielectricii devin permeabili la camp (concentrand si dirijind campul), proprietate descrisa de ct. de material: permitivitate ε si susceptibilitate χ

3. Dielectricii anizotropi au D si E cu orientari diferite, deci cele doua spectre difera. In cel putin trei directii particulare (directiile proprii ale tensorului permitivitatii) E si D sunt totusi coliniare

4. In corpurile polarizate permanent (electreti): Liniile lui **D** sunt contiune si inchise avand directa polarizatiei permanente **P** iar liniile lui **E** sunt deschise, fiind orientate ca **D** in exterior si invers lui **D** si **P** in interior (campul este depolarizant)



Interpretatre microscopica: polarizarea dielectricilor consta in orientarea moleculeleor polare in directii privilegiate. Vectorul polarizatie P indica orientarea si intensitatea acestui fenomen.

Polarizatia temporara dispare/reapare odata cu E iar polarizatia permanenta nu depinde de E.

Teorema refractiei liniilor de camp electric

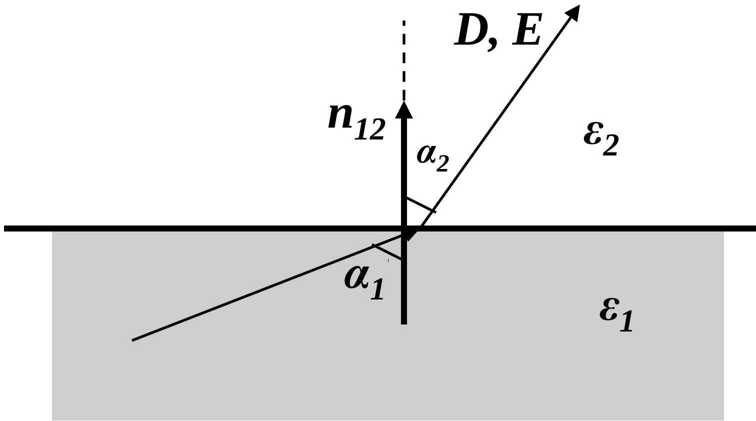
$$\frac{tg\,\alpha_1}{tg\,\alpha_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

1. Pe suprafetele neelectrizate de separatie intre doua corpuri:

$$D_{n1} = D_{n2} \Rightarrow \varepsilon_1 E_{n1} = \varepsilon_2 E_{n2}$$

$$E_{t1} = E_{t2} \Rightarrow \varepsilon_1 E_{n1} / E_{t1} = \varepsilon_2 E_{n2} / E_{t2} \Rightarrow$$

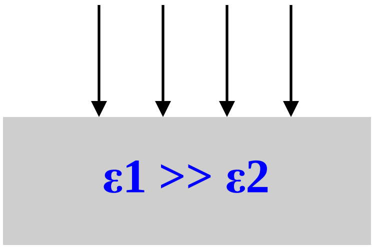
$$\varepsilon_1 / tg\,\alpha_1 = \varepsilon_2 / tg\,\alpha_2 \Rightarrow$$

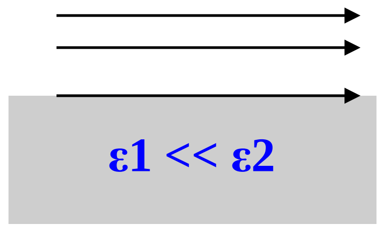


2. Daca $\varepsilon_1 = \varepsilon_2$ linia de camp nu este franta

3. Daca $\varepsilon_1 \rightarrow 0$ ($\varepsilon_1 << \varepsilon_2$) $\alpha_1 \rightarrow 0$ or $\alpha_2 \rightarrow \pi/2$

4. Daca $\varepsilon_1 \rightarrow$ infinit ($\varepsilon_1 >> \varepsilon_2$) $\alpha_2 \rightarrow 0$ or $\alpha_1 \rightarrow \pi/2$

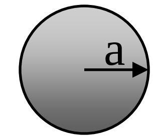


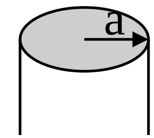


- **Campul evita corpurile de permitivitate scazuta**
- **El este atras de corpurile cu permitivitate inalta**

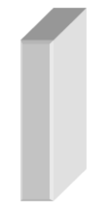
Aplicatii ale legii polarizatiei

1. Calculati si reprezentati grafic campul electric E, D si potentialul V produse de o sfera dielectrica aflata in vid, electrizata uniform cu densitatea de sarcina si pemitivitatea cunoscute. Incercati generalizarea pentru o distributie arbitrara a sarcinii in vid (integralele coulombiene).



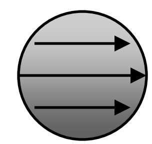


2. Rezolvati problema anterioara inlocuind sfera cu un cilindru

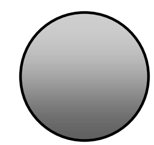


3. Rezolvati problema anterioara pentru cazul unei placi infinit extinse dar de grosime finita

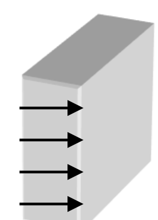
4. Calculati si reprezentati grafic campul electric E, D si potentialul V produse de un cilindru afllata in vid polarizat permanent uniform



5. Determimnati perturbatia unui camp uniform datorata unei sfere dielectrice nepolarizate si neelectrrizate



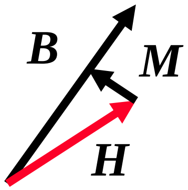
6. Calculati campul si potentialul generate de o placa polarizata uniform



7. Determinati forma globala a legii in cazul unui cilindru cu camp uniform, orientat axial.

Legea legaturii dintrew B si H (a magnetizatiei)

1. Enunt: Inductia magnetica dintr-un punct depinde de intensitatea campului magnetic din acel punct. Forma concreta a dependentei este functie de substanta in care se afla punctul.



2. Forma locala generala:

$$\mathbf{B} = \mathbf{f}(\mathbf{H}), \quad \mathbf{f}: \mathbb{R}^3 \to \mathbb{R}^3$$

caracteristica de magnetizarea

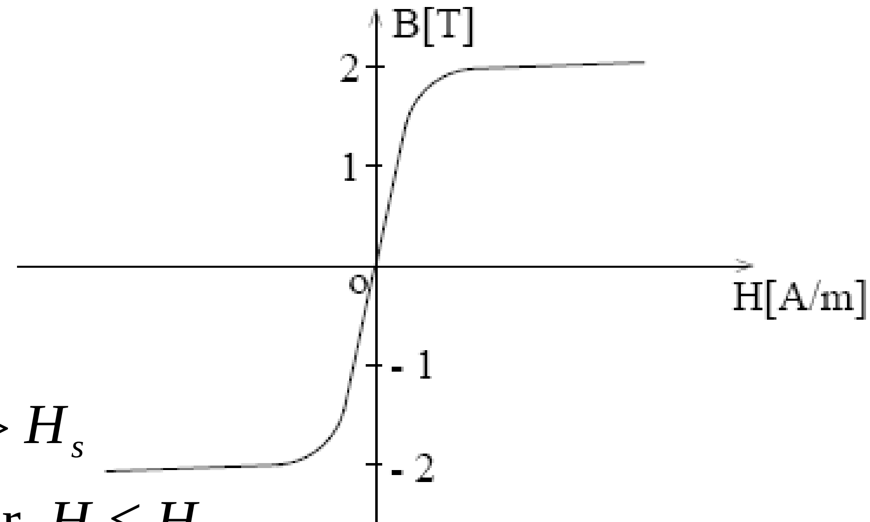
3. Forme particulare:

- in vid (si medii nemagnetice): $\mathbf{B} = \mu_0 \mathbf{H}, \quad \mu_0 = 4\pi \cdot 10^{-7}\, H/m$ – permeabilitatea vidului
- in medii linare si izotrope: $\mathbf{B} = \mu \mathbf{H}, \quad \mu_r = \mu / \mu_0 \Rightarrow \mu = \mu_r \mu_0$
- in in medii anizotrope: $\mathbf{B} = \bar{\bar{\mu}} \mathbf{H}, \quad \bar{\bar{\mu}} = \begin{bmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{21} & \mu_{22} & \mu_{23} \\ \mu_{31} & \mu_{32} & \mu_{33} \end{bmatrix}, \mu_{ij} = \mu_{ji}, \mathbf{HB} = \mathbf{H}\bar{\bar{\mu}}\mathbf{H} > 0$
- in magneti permanenti (model afin): $\mathbf{B} = \bar{\bar{\mu}} \mathbf{H} + \mathbf{I}_p, \mathbf{I}_p = \mu_0 \mathbf{M}_p \Rightarrow \mathbf{B} = \bar{\bar{\mu}} \mathbf{H} + \mu_0 \mathbf{M}_p = \mu_0 (\bar{\bar{\mu}}_r \mathbf{H} + \mathbf{M}_p)$
- in general: $\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) \Rightarrow \mathbf{M} = \mathbf{B}/\mu_0 - \mathbf{H} = \mathbf{M}_t(\mathbf{H}) + \mathbf{M}_p, \mathbf{M}_p = \mathbf{f}(0)/\mu_0$
- in medii liniare: $\mathbf{M}_p = 0, \mathbf{M} = \mathbf{M}_t(\mathbf{H}) = \chi_m \mathbf{H}, \mathbf{B} = \mu_0 (1 + \chi_m) \mathbf{H} \Rightarrow \mu_r = 1 + \chi_m$

4. Magnetizatia: $\mathbf{M} = \mathbf{B}/\mu_0 - \mathbf{H} = \mathbf{M}_t(\mathbf{H}) + \mathbf{M}_p [A/m]$ marime derivata – descrie magnetizarea

5. Semnificatie fizica: legea descrie fenomenele de magnetizarea permanenta – sursa de camp magnetic si magnetizarea temporara, datorita careia campul magnetic este perturbat

Materiale feromagnetice



1. Materiale feromagnetice moi:

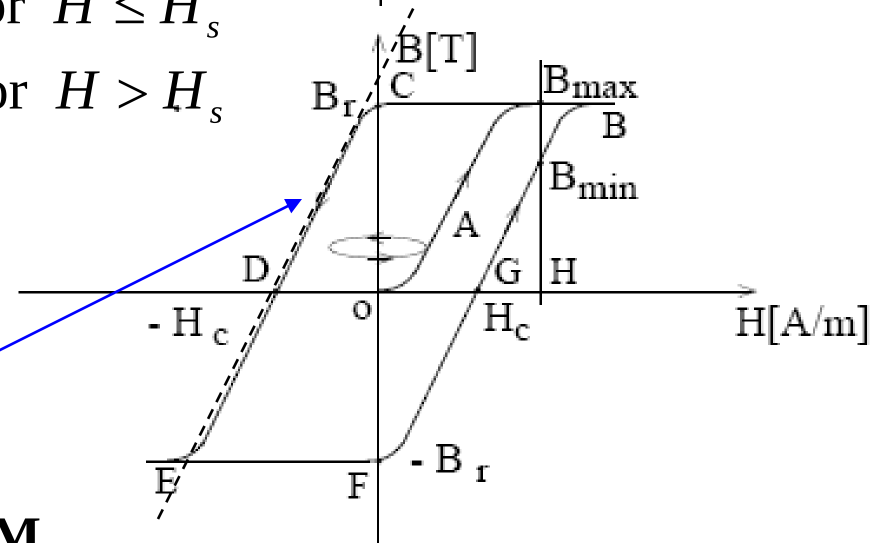
B || H, B = f(H) f:R$_+$→ R caracteristica de magnetizare, de exemplu aproximata liniar pe portiuni:

$$B = f(H) = \begin{cases} \mu_r \mu_0 H, \text{for } H \le H_s \\ \mu_0 (H + (\mu_r - 1) H_s), \text{for } H > H_s \end{cases}$$

$$M = f(H)/\mu_0 - H = \begin{cases} \chi H = (\mu_r - 1) H, \text{for } H \le H_s \\ H_s = (\mu_r - 1) H_s, \text{for } H > H_s \end{cases}$$

$$\mu_r = 100 - 100000, B_s = \mu_r \mu_0 H_s = 0.5...2T$$

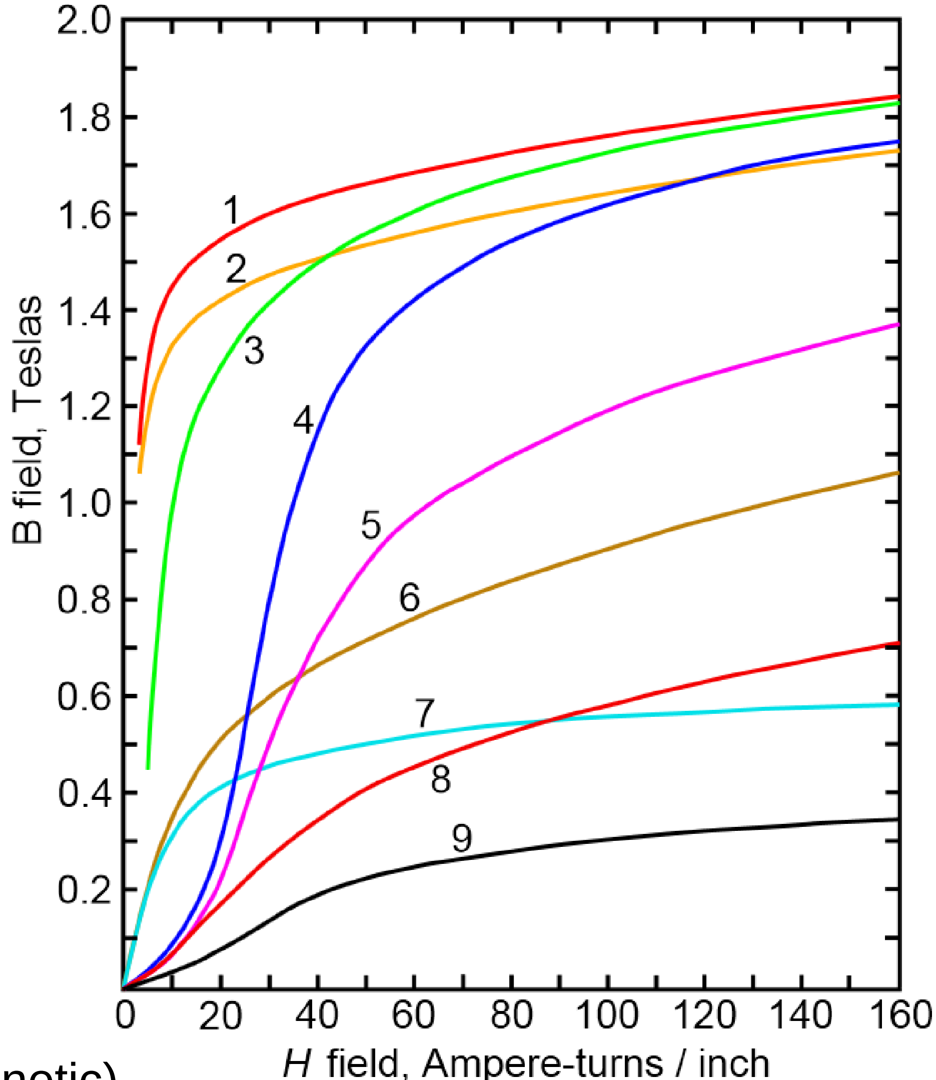
2. Materiale magnetice dure – prezinta fenomenul de histerezis magnaetic (memorie magnetica)- pentru magneti permanenti.



Modelul afin pt cadranul 2: $\mathbf{B} = \mu \mathbf{H} + \mu_0 \mathbf{M}_p$

Materiale feromagnetice moi – caracteristici de magnetizare

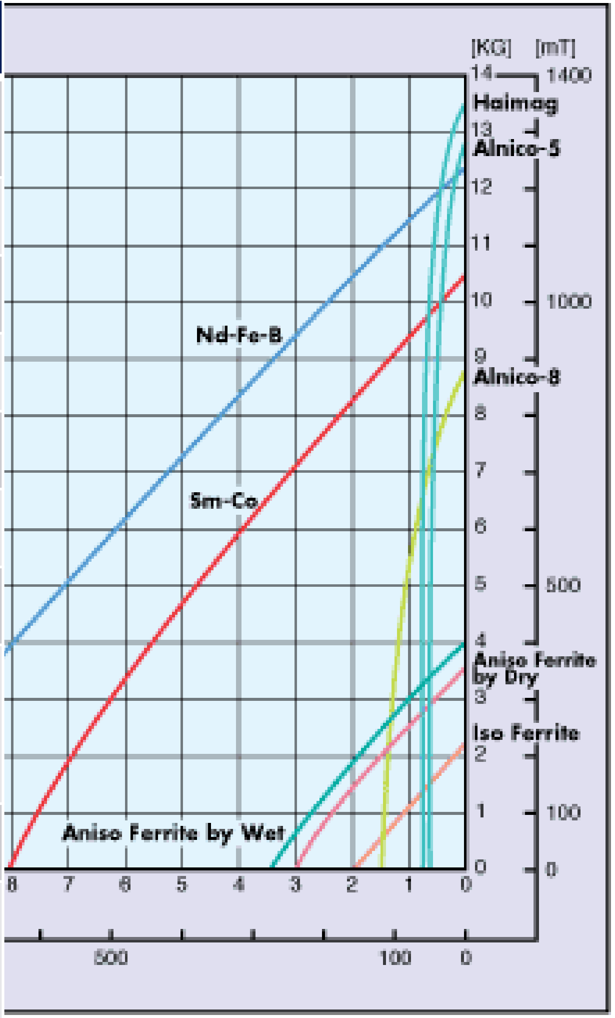
1. Sheet steel,
2. Silicon steel,
3. Cast steel,
4. Tungsten steel,
5. Magnet steel,
6. Cast iron,
7. Nickel,
8. Cobalt,
9. Magnetite



http://en.wikipedia.org/wiki/Saturation_(magnetic)

Caracteristici tipice de histerezis

Type of magnets		Rare Earth		Ferrite			Alnico Magnet		
Item	Unit	Nd-Fe-b	Sm-Co	Isotropic Ba-Ferrite	Aniso- tropic Sr-Ferrite by Dry	Aniso- tropic sr-Ferrite by wet	Alnico-5	Alnico-8	Haimag Alnico-5 col
Brr	[kG]	12.4	10.5	2.2	3.6	4	12.7	8.8	13.5
	(mT)	1,240	1,050	220	360	400	1,270	880	1,350
bHc	[kOe]	11.6	8	1.9	3	3.3	0.65	1.47	0.75
	(kA/m)	923	636	151	238	262	51	117	59
Bhmax	[MGOe]	37	24	1	3	3.8	5.3	5.2	7.3
	(kJ/ÜG)	294.5	191	8	23.9	30.2	42.2	41.4	58.1
Tempe-rature charact-eristic of Br	%/Åé	-0.12	-0.04	-0.18	-0.18	-0.18	-0.02	-0.01	-0.02
Curie point	°C	320	750	460	460	460	850	850	850
Density	g/cÜG	7.4	8.3	4.8	4.8	4.9	7.3	7.3	7.3

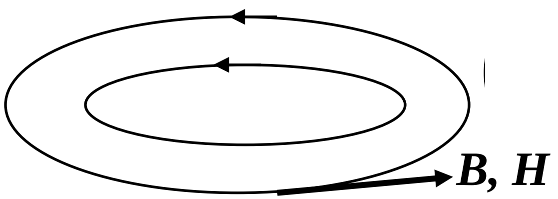


http://www.tmcmagnetics.com/magnet_products.html

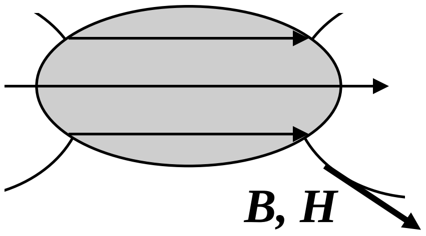
Liniile campului magnetic in corpuri

1. **In vid B asi H au linii comune (este suficient un singur csmp vectorial pentru a descrie campul magnetic in vid)**
2. **Datorita magnetizarii corpurile feromagnetice moi au permeabilitate inaltcon si centreaza si dirijeaza liniile capului magnetic**
3. **In corpurile anizotrope B si H pot avea directii diferite**
4. **In magnetii permanenti:**

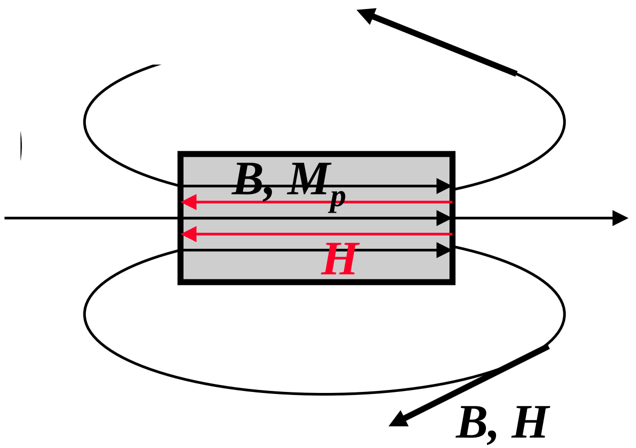
- Liniile lui B sunt continui si inchise, avand directia magnetizatiei permanente M_p
- Liniile lui H sunt curbe descise cu directie comuna cu B in exterior si opusa lui B in interior (se spune ca au camp demagnetizant). Din acest motiv este important cadranul 2 din planul B-H.



In vid



In soft materiale moi



In amgneti permanenti

Teorema refractiei liniilor de camp magnetic

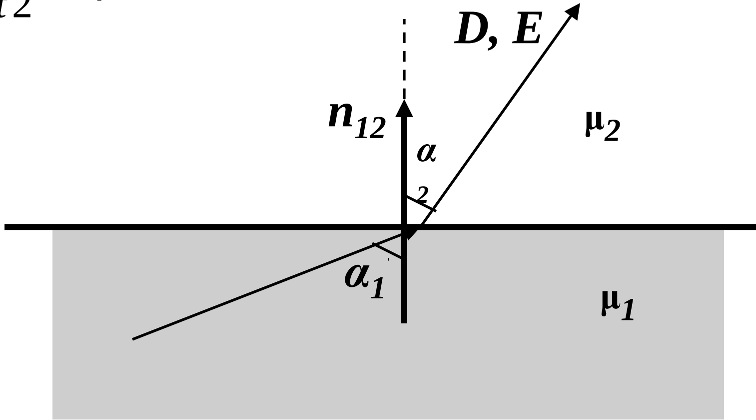
$$\frac{tg\,\alpha_1}{tg\,\alpha_2} = \frac{\mu_1}{\mu_2}$$

1. Pe interfata dintre doua medii:

$$B_{n1} = B_{n2} \Rightarrow \mu_1 H_{n1} = \mu_2 H_{n2}$$

$$H_{t1} = H_{t2} \Rightarrow \mu_1 H_{n1} / H_{t1} = \mu_2 H_{n2} / H_{t2} \Rightarrow$$

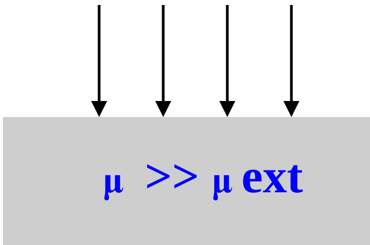
$$\mu_1 / tg\,\alpha_1 = \mu_2 / tg\,\alpha_2 \Rightarrow$$



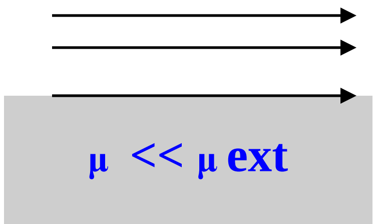
2. Cand μ1= μ 1 campul nu este perturbat (liniile de camp nu se frang)

3. Daca μ 1→ 0 (μ 1 << μ 2) α1→ 0 or α2→ π/2

4. Daca μ 1→ infinit (μ 1 >> μ 2) α2→ 0 or α1→ π/2



Mediu feromagnetic perfect (1/ μ →0, Hint = 0)

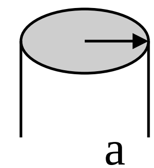


Mediu amagnetic (μ =0, Bint = 0)

- **Campul magnetic evita corpurile de joasa permeabilitate**
- **el este atras de corpurile permeabile**

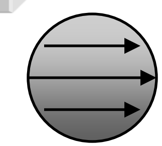
Aplicatii ale legii magnetizatiei

1. Calculati si reprezentati grafic campul magnetic H, B produs de un conductor cilindric afllata in vid, parcurs de un curent uniform cu orientat longitudinal. Incercati generalizarea pentru o distributie neuniforma a curentului.

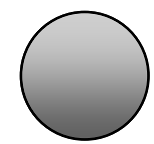


2. Rezolvati problema anterioara pentru cazul unei placi infinit extinse dar de grosime finite

3. Calculati si reprezentati grafic campul magnetic H,B produse de un cilindru aflat in vid si magnetizat permanent uniform. Folositi similitudinea cu campul electric



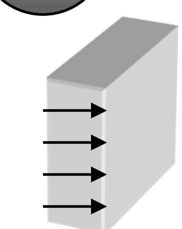
4. Determimnati prin similitudine perturbatia unui camp magnetic uniform datorata unei sfere de permeabilitate cunoscuta aflata in vid.



5. Calculati campul si potentialul generate de o placa magmetizata uniform

6. Determinati forma globala a legii in camp uniform.

7. Aratati de ce campul dintr-o fanta alungita orientata de-a lungul liniilor de camp are aceiasi intensitate Ht ca si campul din corp in schimb inductia Bn se conserva, doar daca fanta este plata si orientata transversal fat de camp. Aratati ca tensiunea pe curba C dintr-un corp nu se modifica daca se practica o fanta vida (tunel) de-a lungul curbei C, iar fluxul de pe o suprafata S nu se modifica daca, se practica o fanta vida in jurul suprafetei S.



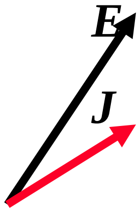
Legea conductiei. Ohm

Enunt: densitatea de curent depinde de campul electric:

1. **Forma locala a legii:** $\mathbf{J} = \mathbf{f}(\mathbf{E})$

2. **Forme particulare:**

- **in vid** $\mathbf{J} = 0$
- **in conductore liniare izotrope** $\mathbf{J} = \sigma\mathbf{E} \Leftrightarrow \mathbf{E} = \rho\mathbf{J}, \text{with } \rho = 1/\sigma$
- **in conductoare liniare anizotrope** $\mathbf{J} = \bar{\sigma}\mathbf{E}$
- **in corpuri cu camp imprimat** $\mathbf{J} = \bar{\bar{\sigma}}(\mathbf{E} + \mathbf{E}_i)$



3. Clasificarea corpurilor: izolante $\sigma=0$; (slabe-, semi-, bune-)conductoare; supraconductoare $\rho = 1/\sigma = 0$. Conductivitatea depinde de temperatura.

Strapungerea izolantilor: Modificare ireversibila in conductor, cand E>Emax

4. Semnificatie fizica: Curentul este datorat campului electric. Campul electric imprimat genereaza camp electric. Fenomen ce descrie cauzele ne-electrice ale campului (chimice, ca in pile si acumulatore, termice, mecanice…)

5. Liniile de camp electric generate de campul imprimat : sunt deschise si se opun campului imprimat Ei in special in absenta curentului

Recapitularea legilor de material

Camp:	Polarizatie	Magnetizatie	Conductie
General	$\mathbf{D} = \mathbf{f}(\mathbf{E})$	$\mathbf{B} = \mathbf{f}(\mathbf{H})$	$\mathbf{J} = \mathbf{f}(\mathbf{E})$
Vid	$\mathbf{D} = \varepsilon_0 \mathbf{E},$	$\mathbf{B} = \mu_0 \mathbf{H}$	$\mathbf{J} = 0$
Liniar Izotropic	$\mathbf{D} = \varepsilon \mathbf{E},$	$\mathbf{B} = \mu \mathbf{H},$	$\mathbf{J} = \sigma \mathbf{E}$
Liniar anizotropic	$\mathbf{D} = \bar{\bar{\varepsilon}} \mathbf{E},$	$\mathbf{B} = \bar{\bar{\mu}} \mathbf{H},$	$\mathbf{J} = \bar{\bar{\sigma}} \mathbf{E}$
Modleul afin	$\mathbf{D} = \bar{\bar{\varepsilon}} \mathbf{E} + \mathbf{P}_p$	$\mathbf{B} = \bar{\bar{\mu}} \mathbf{H} + \mu_0 \mathbf{M}_p$	$\mathbf{J} = \bar{\bar{\sigma}} (\mathbf{E} + \mathbf{E}_i)$
Medii neliniare	$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \mathbf{E} + \mathbf{P}_t(\mathbf{E}) + \mathbf{P}_p$	$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0(\mathbf{H} + \mathbf{M}_t(\mathbf{H}) + \mathbf{M}_p)$	$\mathbf{J} = \bar{\bar{\sigma}} (\mathbf{E} + \mathbf{E}_i(\mathbf{E}))$
Refractia liniilor de camp	$\frac{tg\alpha_1}{tg\alpha_2} = \frac{\varepsilon_1}{\varepsilon_2}$	$\frac{tg\alpha_1}{tg\alpha_2} = \frac{\mu_1}{\mu_2}$	$\frac{tg\alpha_1}{tg\alpha_2} = \frac{\sigma_1}{\sigma_2}$

Concluzii privind legile de material

- Fiecare substanta are propria comportare din punct de vedere dielectric, magnetic si al conductiei. Formele particulare ale relatiilor de material sunt **relatii constitutive ale electromagnetismului.** Ele se determina prin proceduri experimentale dedicate, de modelare a materialeleor.
- In general relatiile constitutive sunt descrise **de functii vectoriale neliniare** de variabila vectoriala care se aproximeaza suficient de bine de **modele afine**, in care caractersticile de material sunt descrise de un tensor si un camp vectorial. Multe medii admit modele de material si mai simple, la care tensorul are valorile proprii egale si se reduce deci la un simplu scalar iar componentele permanete sunt nule. Se obtine astfel **modelul liniar izotrop**, in care cei doi vectoiri din legile de material sunt coliniari si proportionali, iar substanta este caracterizata complet doar de trei constante de material: permitivitate, permeabilitate si conductivitate. In mediile liniare **este valabil principiul superpozitiei.** Mediile liniare cu comportare similara vidului se numesc: nepolarizabile, nemagnetizabile si izolante.
- Starea electromagnetica a corpurilor este descrisa de **marimile primitive** ρ si $\mathbf{J}$ dar si de marimile derivate: polarizatia $\mathbf{P}$ si magnetizatia $\mathbf{M}$.
- Relatiile $\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{M}$ si $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$ pot fi folosite pentru a defini inductia magnetica si respectiv pe cea electrica, caz in care acestea devin **marimi derivate**, iar $\mathbf{P}$ si $\mathbf{M}$ vor fi **marimi primitive**. O astfel de alegere corespunde unei teorii electromagnetice echivalente, in care campurile sunt caracterizare de doua marimi primitive $\mathbf{E}$ si $\mathbf{H}$ iar corpurie de patru marimi primitive: ρ**,** $\mathbf{J}$**,** $\mathbf{P}$ si $\mathbf{M}$**.** Marimile globale asociate lui $\mathbf{P}$ si $\mathbf{M}$ se obtin prin integrarea lor pe volum si definesc **momentul electric** $\mathbf{p}[\mathrm{Cm}]$**, si momentul magnetic** $\mathbf{m}[\mathrm{Am}^2]$**.**

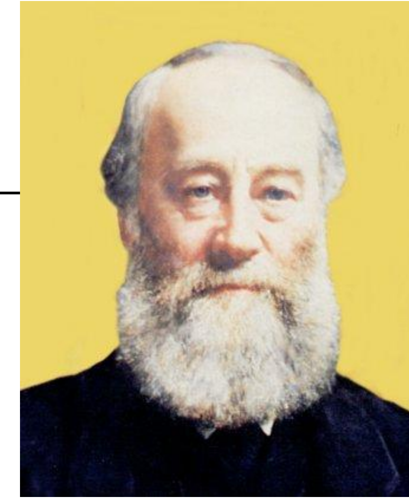
Valori ale constantelor de material

Material	Resistivity (nΩ·m)	Density (g/cm³)	Resistivity-density product (nΩ·m·g/cm³)
Aluminium	26.50	2.70	72
Copper	16.78	8.96	150
Silver	15.87	10.49	166
Gold	22.14	19.30	427
Iron	96.1	7.874	757

Medium	Susceptibility χ_m	Permeability μ [H/m]	Relative Permeability μ/μ_0
Metglas		1.25×10^{-1}	1,000,000[6]
Mu-metal			50,000[9]
Permalloy		1.0×10^{-2}	8,000[8]
Ferrite (nickel zinc)		2.0×10^{-5} – 8.0×10^{-4}	16–640
Steel		8.75×10^{-4}	100[8]
Nickel		1.25×10^{-4}	100[8] – 600
Superconductors	−1	0	0

Material	*Relative permittivity* E_r
Teflon	2.1
Polyethylene	2.25
Polyimide	3.4
Polypropylene	2.2–2.36
Polystyrene	2.4–2.7
Carbon disulfide	2.6
Paper	3.85
Silicon dioxide	3.9 [3]
Concrete	4.5
Pyrex (Glass)	4.7 (3.7–10)
Rubber	7
Diamond	5.5–10
Water	88, 80.1, 55.3 (0, 20, 100, °C)

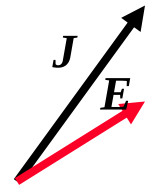
Legea transferului de energie - Joule



1. Enunt: Campul electromagnetic transfera corpurilor o putere a carei densiatae de volum este**:**

2. Forma local a legii:

$$p = \mathbf{E} \cdot \mathbf{J} \quad [\mathrm{W/m^3}]$$



3. Forme particulare

- in conductoare limiare: $$p = \mathbf{E} \cdot \mathbf{J} = \sigma E^2 = \rho J^2 \geq 0$$

- in conductoare neliniare: $$p = \mathbf{E} \cdot \mathbf{J} = \sigma(\mathbf{E} + \mathbf{E}_i)\mathbf{E} = \sigma \mathbf{E}^2 + \sigma \mathbf{E}\mathbf{E}_i =$$

$$(\rho \mathbf{J} - \mathbf{E}_i)\mathbf{J} = \rho \mathbf{J}^2 - \mathbf{E}_i \mathbf{J}$$

4. Semnificatie fizica:

- Legea descrie fenomenul electro-termic de incalzire a corpurilor parcurse de curent (in stare electrocinetica)
- Transferul de energie in conductoare liniare are un caracter ireversibil

5. Definitia densitatii de volum a puterii:

$$p = \lim_{\Delta V \to 0} \frac{\Delta P}{\Delta V} = \frac{dP}{dV} \text{ cu } P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

6. Forma integrala a legii:

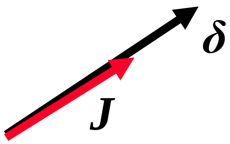
$$P = \int_\Omega p dV = \int_\Omega \mathbf{J} \cdot \mathbf{E} dV = ui \Rightarrow W = \int_0^T P dt = uiT$$

Legea transferului de masa (a electrolizei) - Faraday

1.Enunt: in conductie are loc un transfer de masa cu densitatea fluxului de masa $\boldsymbol{\delta}$:

2. Forma locala a legii $\boxed{\boldsymbol{\delta} = k\mathbf{J}}$ $[\mathrm{kg/m^2 s}]$.

$$k = \begin{cases} \approx 0; \text{in metale} \\ \dfrac{M}{Fz}; \text{in electroliti} \end{cases}$$



3. Forma globala – legea electrolizei

$$Q_m = \int_s \boldsymbol{\delta} d\mathbf{A} = \int_s k\mathbf{J}\, d\mathbf{A}$$

$$m = \int_{t1}^{t2} Q_m dt = \int_{t1}^{t2}\int_s k\mathbf{J}\, d\mathbf{A} dt =$$

$$= k\int_{t1}^{t2}\int_s \mathbf{J}\, d\mathbf{A} dt = k\int_{t1}^{t2} i dt = \boxed{kIt = m}$$

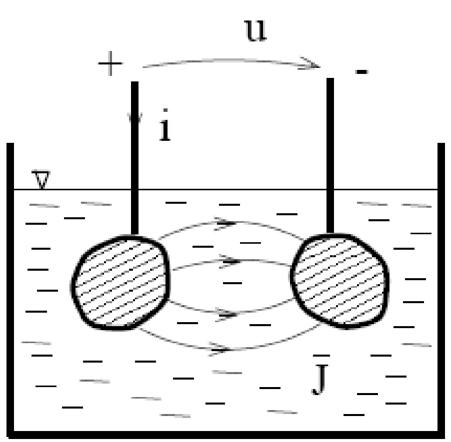
$\mathrm{k = coeficientul\ electrochimic}$

$\mathrm{F} = 96490\, C/mol$ - Constanta lui Faraday

$\mathrm{M = masa\ molara}$

$\mathrm{z = valenta}$

3. Semnificatie fizica: Legea descrie transferul de masa, efect al curentului electric.



4. Definita densitatii fluxului de masa:

$$\boldsymbol{\delta} = \mathbf{n} \lim_{\Delta A \to 0} \frac{\Delta Q_m}{\Delta A} = \mathbf{n} \frac{dQ_m}{dA} [kg/s \cdot m^2]$$

$$Q_m = \lim_{\Delta t \to 0} \frac{\Delta m}{\Delta t} = \frac{dm}{dt} [kg/s] \text{ - debit masic}$$

Aplicatii, exercitii si probleme

1. In ce conditii este valabila forma globala a legii polarizatiei: $\psi = C u$?
2. In ce conditii este valabila forma global a legii magnetizatiei: $\varphi = \Lambda U_m$?
3. In ce conditii este valabila forma global a legii conductiei: $i = G u$?
4. In ce conditii este valabila forma global a legii transferului de energie $p = ui$?
5. Cum arata expresiile legilor de material in cazul mediilor liniare/neliniare, izotrope/ anizotrope, omogene/neomogene? Incercati cat mai multe combinatii.
6. Cum se obtine cea mai buna aproximare afina a unei caracteristici neliniare de material.
7. Cautati modele pentru materilele feromagnetice moi anizotrope: expresii matematice ale caracteristicii de magnetizare B-H
8. Cautati modele pentru fenomenul de histerezis din materialele magnetice dure: ecuatii matematice care sa descrie relatia B-H
9. Poate depinde denstatea de curent de campul magnetic: J(E, B) ? In ce conditii ?
10. Identificati relatiile constitutive pentru toate materialele din camera in care va aflati
11. De ce F=Ae , constanta lui Faraday supar nr. lui Avogadro da chiar sarcina electronului ?
12. Imaginati-va noua experimente care evidentiaza cele noua fenomene fundamentale ale electromagnetismului.

Concluzii privind legile de transfer

Transfer	Energie	Masa
Local	$p = \mathbf{E} \cdot \mathbf{J} \quad [\mathrm{W/m^3}]$	$\boldsymbol{\delta} = k\mathbf{J} \quad [\mathrm{kg/m^2s}]$
Global	$P = ui \quad [\mathrm{W}]$	$m = kit \quad [\mathrm{kg}]$

• Legile generale impreuna cu cele de material alcatuiesc un sistem complet din punct de vedere matematic. Ele descriu cauzele campului electromagnetic dar nu sunt insa complete din punct de vedere fizic, deoarece nu descriu si efectele acestui camp. Nici o lege din cele generale sau de material nu contin marimi fizice comune si altor stiinte fizice.

• **Legile de transfer descriu efecte ale campului elctromagnetic,** completand astfel sistemul legilor si din punct de vedere fizic. Ele asigura legatura cu alte discipline fizice.

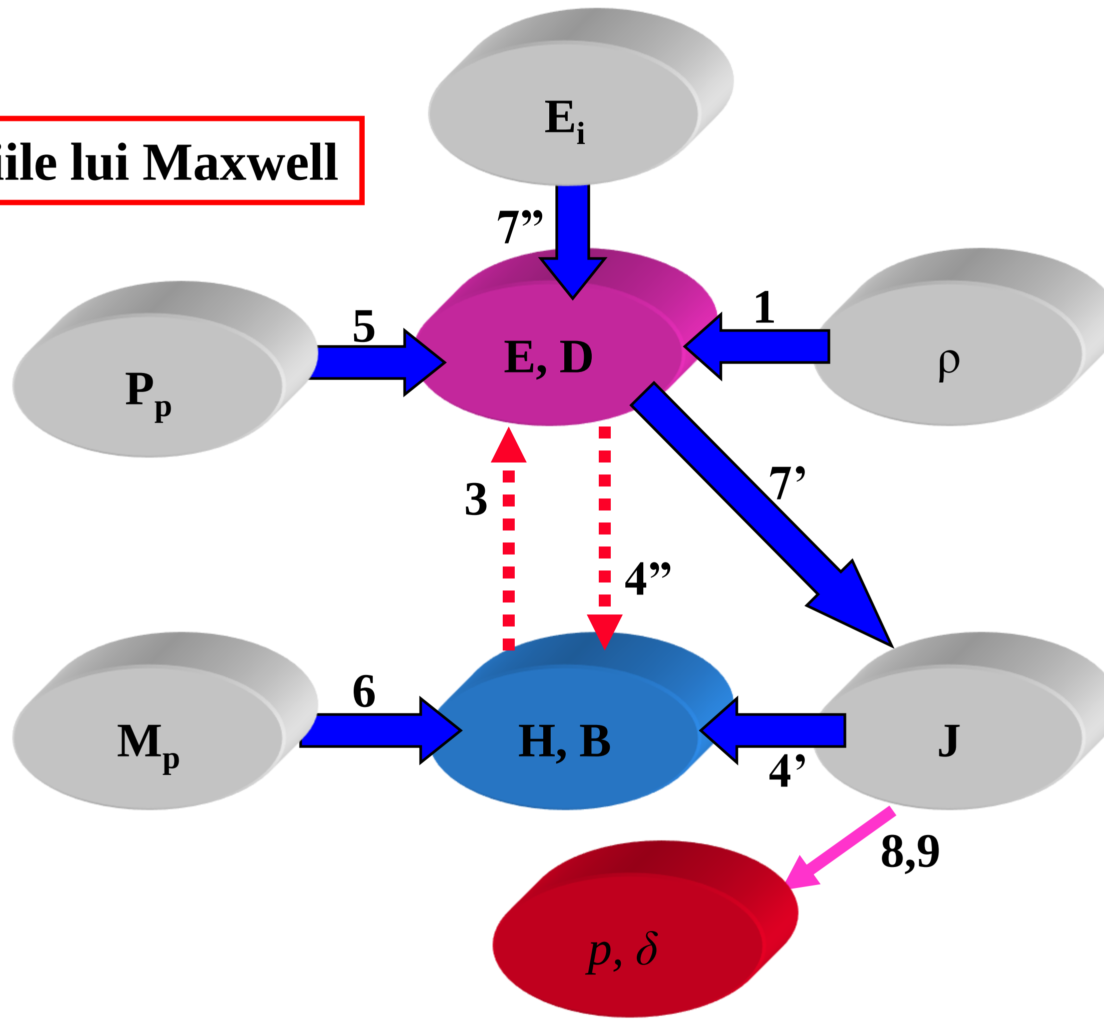
• Legile generale se exprima in general in forma globala, in timp ce legile de material si cele de transfer se exprima in forma locala. Celelalte forme sunt consecinte particulare, deci **teoreme ale electromagnetismului.**

Diagrama relatiilor cauzale si a fenomenelor el-mg fundamentale

Forma locala a legilor el-mg:

Ecuatiile lui Maxwell

1. $\nabla \cdot \mathbf{D} = \rho$
2. $\nabla \cdot \mathbf{B} = 0$
3. $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$
4. $\nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$
5. $\mathbf{D} = \varepsilon\ \mathbf{E} + \mathbf{P}_p$
6. $\mathbf{B} = \mu\ \mathbf{H} + \mu_0 \mathbf{M}_p$
7. $\mathbf{J} = \sigma(\mathbf{E} + \mathbf{E}_i)$
8. $p = \mathbf{EJ}$
9. $\boldsymbol{\delta} = k\mathbf{J}$



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