A non-overlapping domain decomposition approach for magneto-quasistatics

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***Abstract*—Simulating land mine detection scenarios poses a challenge regarding the discretisation of the metal detector and the much smaller metal parts of the mine. Here, a domain decomposition method with Lagrange multipliers is presented. This allows a magneto-quasistatic simulation with locally very fine mesh around the land mine. This approach solves only one system of equations for the fine and coarse part together.**

Keywords—domain decomposition, finite integration technique, magneto-quasistatics, land mine detection

I. Introduction

Metal detectors are one of the best land mine detection device due to the good percentage of located mines to non-located ones, the easy handling and the low price. However the metal detectors react on every metal piece in the soil regardless if it is a mine or not. The rate between located mine and sounding signal is between 1/100 and 1/1000 [1].

To reduce this false alarm rate it is proposed to precalculate signatures of standard mines, save them in a database and compare the measured signatures with the data from the database. For the calculation of mine signatures different resolutions for the metal detector and the small metal parts of the land mine, which are about two orders of magnitude smaller, are mandatory. The discretisation of such a problem with standard methods leads either to a huge amount of mesh points, which results in unacceptable time consuming computations or the required resolution is not reached at all. In this paper the calculation domain is decomposed into two parts: the land mine is discretised with a fine mesh while a coarser mesh is chosen for the rest. Both subdomains are coupled with interface conditions, which results in a new system of linear equations for the whole calculation domain.

II. Magneto-quasistatic equations

Metal detectors work either with time-harmonic or pulsed signals for the transmitting coil. The considerations of this paper are restricted to the time-harmonic excitation. In this case the displacement current density is much smaller than the total current density such that wave propagation effects can be neglected and the magneto-quasistatic approach can be applied. With the source current density $\boldsymbol{J_s}$ the equation for the magnetic vector potential $\boldsymbol{A}$ is given as

$$\mathrm{curl}\frac{1}{\mu}\mathrm{curl}\boldsymbol{A} + i\omega\sigma\boldsymbol{A} = \boldsymbol{J_s}, \tag{1}$$

where μ and σ are permeability and conductivity, respectively. The angular frequency is denoted by ω and the imaginary unit by i.

In the Finte Integration Technique (FIT) [2], [3] the electromagnetic field quantities are represented on a dual grid pair. The analytical grad, div and curl operators are transferred to the primary and dual grid operators $\boldsymbol{G}$, $\widetilde{\boldsymbol{G}}$, $\boldsymbol{S}$, $\widetilde{\boldsymbol{S}}$, $\boldsymbol{C}$ and $\widetilde{\boldsymbol{C}}$, respectively. It can be shown that the analytical properties, a curl field is free of sources and a gradient field is irrotational, are also valid for these operators, i.e. $\boldsymbol{SC}$=$\widetilde{\boldsymbol{S}}\widetilde{\boldsymbol{C}}$=0 and $\boldsymbol{CG}$=0. The material relations are transferred approximately with the help of the material matrices for permeability $\boldsymbol{M_{\mu^{-1}}}$ and conductivity $\boldsymbol{M_\sigma}$. Let $\widehat{\mathrm{a}}$ denote the discrete magnetic vector potential, which is located on the primary grid edges and $\widehat{\widehat{\mathrm{j}}}$ the discrete vector of source currents through the dual grid facets. Then, in the framework of FIT, equation (1) leads to

$$(\widetilde{\boldsymbol{C}}\boldsymbol{M_{\mu^{-1}}}\boldsymbol{C} + i\omega\boldsymbol{M_\sigma})\widehat{\mathrm{a}} = \widehat{\widehat{\mathrm{j}}}. \tag{2}$$

The system matrix $\boldsymbol{K}$ is defined as

$$\boldsymbol{K} := \widetilde{\boldsymbol{C}}\boldsymbol{M_{\mu^{-1}}}\boldsymbol{C} + i\omega\boldsymbol{M_\sigma} \tag{3}$$

for further considerations.

III. Domain Decomposition approach

The large difference in size of the metal detector (about 30 cm) and the metal parts in modern land mines (often only few millimeters) as well as the distance between both, which is several magnitudes greater than the metal parts, affords an efficient discretisation strategy. Here, a domain decomposition scheme is presented, where the computational domain is subdivided in non-overlapping subdomains. The same approach has been applied in [4], for the calculation of low-frequency electric current densities in high resolution 3D human anatomy models. A fine mesh is applied to the first subdomain containing the small metallic piece and operators referring to this subdomain are denoted with the subscript f. The rest of the computational domain, where the detector coil is located, is discretised with a coarser mesh and the operators will be denoted with the subscript c. For both subdomains the system matrices $\boldsymbol{K_c}$ and $\boldsymbol{K_f}$ are constructed separately.

The domains are non-overlapping and have a common interface Γ. The grids of the subdomains are non-matching, i.e. coarse grid points on Γ are not necessarily fine grid points on Γ, too. On the interface Γ the magnetic vector potential and the normal component of the current have to be continuous. The grid points on the common interface Γ are determined by the selection operators $\boldsymbol{Q_c}$ and $\boldsymbol{Q_f}$, that is

$$\widehat{\mathbf{a}}_{c,\Gamma} = \boldsymbol{Q_c}\widehat{\mathbf{a}}_c, \quad (4)$$
$$\widehat{\mathbf{a}}_{f,\Gamma} = \boldsymbol{Q_f}\widehat{\mathbf{a}}_f. \quad (5)$$

Since both grids are non-matching a prolongation operator $\boldsymbol{B} : \Gamma_c \rightarrow \Gamma_f$ is needed on the interface. Here a bilinear interpolation is chosen to calculate the magnetic vector potential of the fine grid points at the interface. Then the interface condition for the magnetic vector potential reads as:

$$\boldsymbol{B}\widehat{\mathbf{a}}_{c,\Gamma} - \widehat{\mathbf{a}}_{f,\Gamma} = 0, \quad (6)$$
$$\boldsymbol{B}\boldsymbol{Q_c}\widehat{\mathbf{a}}_c - \boldsymbol{Q_f}\widehat{\mathbf{a}}_f = 0. \quad (7)$$

The transpose $\boldsymbol{B^T} : \Gamma_f \rightarrow \Gamma_c$ is applied for the calculation of the normal component of the currents in the coarse grid points of Γ. Both system matrices and the interface conditions are assembled in the following system of linear equations

$$\begin{bmatrix} \boldsymbol{K_c} & 0 & \boldsymbol{Q_c^T B^T} \\ 0 & \boldsymbol{K_f} & -\boldsymbol{Q_f^T} \\ \boldsymbol{BQ_c} & -\boldsymbol{Q_f} & 0 \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{a}}_c \\ \widehat{\mathbf{a}}_f \\ \lambda \end{bmatrix} = \begin{bmatrix} \widehat{\widehat{\mathbf{j}}}_{s,c} \\ \widehat{\widehat{\mathbf{j}}}_{s,f} \\ 0 \end{bmatrix}, \quad (8)$$

where λ denotes the vector of Lagrange multipliers.

The system above represents a saddle-point problem, which can be solved by various algorithms as suggested in [5]. Common iterative methods for such problems are the Uzawa algorithm and the Arrow-Hurwicz algorithm. A major disadvantage of both algorithms is the slow convergence rate. Another possibility is the application of a Krylov-subspace solver like BICG or MINRES together with an efficient preconditioner. The preconditioner is often based on an approximation of the Schur complement $\boldsymbol{S(D)}$ defined by

$$\boldsymbol{S(D)} := \boldsymbol{BQ_cD_c^{-1}Q_c^TB^T} - \boldsymbol{Q_fD_f^{-1}Q_f^T}, \quad (9)$$

which consists of an approximation of the system matrices $\boldsymbol{D_c}$=diag($\boldsymbol{K_c}$) and $\boldsymbol{D_f}$=diag($\boldsymbol{K_f}$). The following preconditioner $\boldsymbol{P(D)}$ is proposed in [6]:

$$\boldsymbol{P(D)} = [\boldsymbol{D_c^{-1}}, \boldsymbol{D_f^{-1}}, \boldsymbol{S^{-1}}]. \quad (10)$$

It turned out that diag($\boldsymbol{P(D)}$) already presents a good preconditioner, the construction of which requires much less memory compared to $\boldsymbol{P(D)}$.Yet, the residual of the algorithm using $\boldsymbol{P(D)}$ itself is smaller.

IV. RESULTS

As model problem a circular transmitting coil with the source current of 1 A and frequency of 2400 Hz represents the metal detector. It is located above a metallic sphere with conductivity of $1.7 * 10^7$ S/m. The receiving coil is modelled as a difference coil at the same position as the transmitting coil. For this model both coils are assumed to be infinitely thin. Figure 1 shows the electric field $\boldsymbol{E}$ for this geometry. The box shows the size of the subdomain with fine discretisation, which is placed around the aluminium sphere. A convergence study and further examples will be given in the full paper.

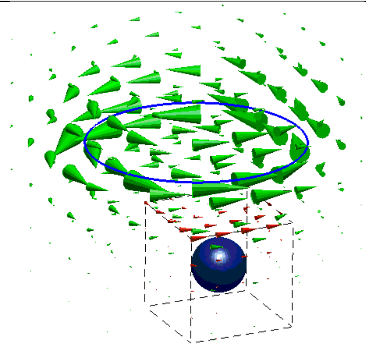

Fig. 1. Electric field $\boldsymbol{E}$ of a transmitting coil above a metallic sphere.

V. Conclusion

In this paper a domain decomposition method with Lagrange multipliers for magneto-quasistatics has been presented. By subdivision of the computational domain an efficient discretisation of land mine detection scenarios may be obtained. For the resulting saddle-point problem two different solution methods have been tested. The proposed approach has been applied to the simulation of a model problem.

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