Automatic partitioning for multirate methods

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***Abstract*—Transient analysis is a very important circuit simulation technique. The circuit model, which is a system of differential-algebraic equations, is solved for a given initial condition using numerical time integration techniques. Multirate methods are very efficient if the activity of the circuit model is not uniform. Automatic partitioning is needed to spllit the model in two parts such that the maximal speed-up factor is achieved.**

Keywords—automatic partitioning, circuit simulation, differential-algebraic equations, multirate, numerical time integration, transient analysis.

I. Introduction

Analogue electrical circuits are usually modelled by differential-algebraic equations of the following type:

$$\frac{d}{dt}\left[\mathbf{q}(t,\mathbf{x})\right] + \mathbf{j}(t,\mathbf{x}) = \mathbf{0}, \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^d$ represents the state of the circuit. A common analysis is the transient analysis, which computes the solution $\mathbf{x}(t)$ of this non-linear DAE along the time interval $[0,T]$ for a given initial state. Often, parts of electrical circuits have multirate behaviour, which means that some variables are slowly varying, compared to other variables.

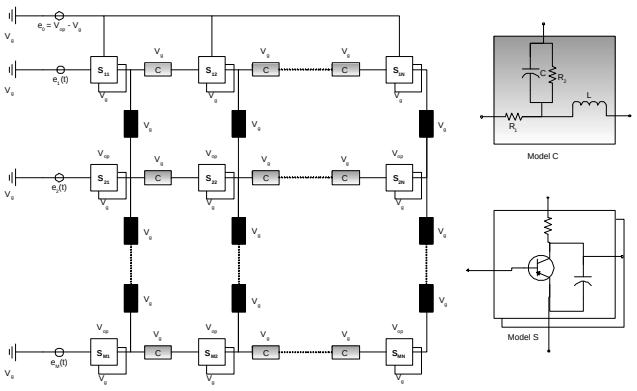

Fig. 1. Circuit diagram of test example.

Figure 1 shows an example, which is a scalable circuit with 5×5 inverters. The subcircuits are connected with linear filters which were chosen such that 3 subcircuits are active and nearly decoupled from the other subcircuits. Let $\{h_1, \ldots, h_d\}$ be the required time-steps per unknown and $h_{\max} = \max\{h_1, \ldots, h_d\}$. Figure 2 shows the relative time-steps $\frac{h_i}{h_{\max}}$ per unknown. Clearly the most variables are not active, so it is very attractive to partition the model and use multirate methods [1], [2].

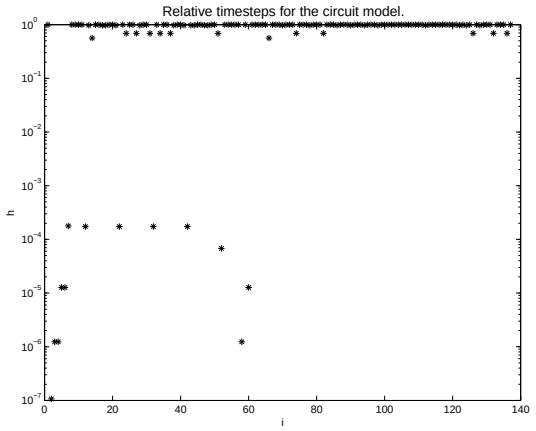

Fig. 2. Relative time-steps per unknown for test example.

II. Partition of the system

Here we will limit ourselves to partition the circuit into two complementary parts. Multirate methods integrate both parts with different time-steps H and h. For circuits with multirate behaviour, like the previous example, the multirate factor $q = \frac{H}{h}$ can be a large number.

For a multirate method it is necessary to partition the variables and equations into an active (A) and a latent (L) part. This can be done by the user or automatically. Let $\mathbf{B}_A \in \mathbb{R}^{d_A \times d}$ and $\mathbf{B}_L \in \mathbb{R}^{d_L \times d}$ with $d_A + d_L = d$ be the permutation mappings, which satisfy $\mathbf{B}_A\mathbf{B}_A^T = \mathbf{I}, \mathbf{B}_L\mathbf{B}_L^T = \mathbf{I}, \mathbf{B}_A\mathbf{B}_L^T = \mathbf{O}, \mathbf{B}_L\mathbf{B}_A^T = \mathbf{O}$. Then the variables and functions can be split in active (A) and latent (L) parts:

$$\begin{aligned} \mathbf{x} &= \mathbf{B}_A^T\mathbf{x}_A + \mathbf{B}_L^T\mathbf{x}_L, \\ \mathbf{q}(t,\mathbf{x}) &= \mathbf{B}_A^T\mathbf{q}_A(t,\mathbf{B}_A\mathbf{x},\mathbf{B}_L\mathbf{x}) + \mathbf{B}_L^T\mathbf{q}_L(t,\mathbf{B}_A\mathbf{x},\mathbf{B}_L\mathbf{x}), \\ \mathbf{j}(t,\mathbf{x}) &= \mathbf{B}_A^T\mathbf{j}_A(t,\mathbf{B}_A\mathbf{x},\mathbf{B}_L\mathbf{x}) + \mathbf{B}_L^T\mathbf{j}_L(t,\mathbf{B}_A\mathbf{x},\mathbf{B}_L\mathbf{x}). \end{aligned} \tag{2}$$

Now equation (1) is equivalent to the following partitioned system:

$$\frac{d}{dt}\left[\mathbf{q}_A(t,\mathbf{x}_A,\mathbf{x}_L)\right] + \mathbf{j}_A(t,\mathbf{x}_A,\mathbf{x}_L) = \mathbf{0}, \tag{3}$$

$$\frac{d}{dt}\left[\mathbf{q}_L(t,\mathbf{x}_A,\mathbf{x}_L)\right]+\mathbf{j}_L(t,\mathbf{x}_A,\mathbf{x}_L) \quad = \quad \mathbf{0}. \qquad (4)$$

For DAEs the partition should be properly chosen such that the parts (3) and (4) are also solvable. Furthermore it is a nice property if also the stability and index are preserved.

III. Optimization of speed-up factor

Partitioning is an important attribute for multirate methods. Together with the time-steps it can be used to control the local discretization error. Consider a multirate method using a fixed partition and fixed stepsizes H, h. These steps are chosen such that the results are as accurate as a singlerate method using fixed step h_s. Although h can be smaller than h_s, we assume that $\frac{h}{h_s} \approx 1$ is nearly independent of the partition. Then it can be derived that the speed-up factor satisfies approximately

$$S = \frac{1}{\frac{1}{q}+E}, \qquad (5)$$

where $E \approx \left(\frac{d_A}{d}\right)^{\alpha}$ is the workload ratio. For an optimal partition this speed-up factor S will achieve its maximum.

Because h mainly depends on the error of the most active element, it is nearly independent of the partition. Thus the multirate factor $q = \frac{H}{h}$ only can be increased by maximizing H. We know [2] that the step H satisfies: $H = \min\{H_C, H_I\}$, where H_C is the step required by the discretization error for the slow part and H_I is the step required by the interpolation error for the active part. So, increasing q can only be done by increasing H_C or H_I. The step H_C can only be increased if the fastest latent variable which is responsible for H_C is converted to the active part. The step H_I can be increased in two manners. First, the responsible active variable can be converted to the latent part, which clearly increases H_I. Second, some latent interpolated variables which are coupled to our dominant active variable can be converted to the active part, which also increases H_I. The workload ratio E can be decreased by reducing the size of the refinement part. Because both q and E depend on the partition, an optimal partition requires the solution of a discrete optimization problem, which could be very complex for large d.

IV. Heuristics

From (5) it is clear that S can be increased by increasing q or by decreasing E. An easier way to maximize S is to maximize q and minimize E independently. Our heuristic approach iteratively maximizes q and minimizes E. It will (partly) perform the next four steps in succession.,

A Make the latent variable which is responsible for H_C active;

B Make the active variable which is responsible for H_I latent;

C Make a subset of latent variables which is coupled to the previous active variable active;

D Make a subset of active variable latent in order to decrease E.

All steps lead to transitions of only a few variables. One could start with a singlerate method but also from a given partition. Afterwards it should be dynamically updated based on the local error estimates.

References

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