Hierarchical Mixed Multirating in Circuit Simulation

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***Abstract*—In most applications large integrated circuits comprise subcircuits of different functionality causing heterogeneous transient behaviour. Multirate methods exploit local latency by using different stepsizes according to the subcircuits' activity levels at each time point. Following the idea of mixed multirate for ordinary differential equations a hierarchical ROW-based multirate method that can deal with an arbitrary amount of subcircuits is developed.**

Keywords—Circuit simulation, DAEs, mixed multirate ROW schemes, partitioned equations, B-series

I. Modular Modelling

Large electrical circuits are usually modelled in a modular way. Several subcircuits of different functionality are developed separately and glued together. Therefore, each sub-unit has to be equiped with junctions by which the unit communicates with its outside world. This communication is done in terms of currents leaving the unit at the according terminals or *pins*.

To simulate the transient behaviour, network equations that model the electrical circuits have to be generated. Modified nodal analysis (MNA) is the method of choice in professional simulator packages [1]. Applying charge-oriented MNA to a single circuit the quantities x and y refer the node potentials and currents through flux and voltage sources and the charges and fluxes, respectively. To describe the composition of r subcircuits we introduce additional unknowns x_{ext}, which basically are the *pin currents*. Then the network equations of the the overall system are given by

$$\left.\begin{aligned} 0&=\mathcal{A}_\lambda \dot{y}_\lambda + f_\lambda(x_\lambda,t) + \mathcal{A}_{\lambda,\text{ext}} x_{\text{ext}} \\ 0&=y_\lambda - g_\lambda(x_\lambda) \end{aligned}\right\} \text{for } \lambda = 1,\cdots,r$$

$$0 = \mathcal{A}_{\text{ext},1} x_1 + \cdots + \mathcal{A}_{\text{ext},r} x_r \ . \tag{1}$$

The compliance of conditions to the network's topology, comparable to the ones that are known for uniform circuits [2], guarantee an index 1 of the coupled DAE problem (1) [3].

II. Hierarchical Mixed Multirate

Mixed multirate for ODEs [4] is a ROW based scheme that deals with two different levels of activity, but schemes for more subblocks are required in industrial circuit simulation.

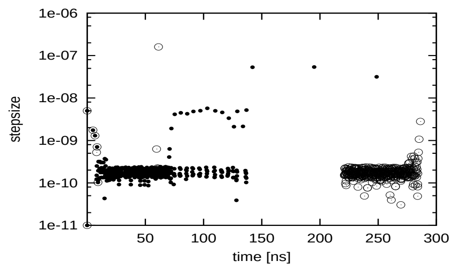

Fig. 1. Hierarchical mixed multirate: stepsizes for inverter chain, inverters $1-100$ ($\bullet$) and $401-500$ ($\circ$)

Starting from a synchronous system, mixed multirate computes in one *compound-step* approximations to the latent part along with the active one using stepsizes $\mathcal{H}_L$ and $\mathcal{H}_A(\ll \mathcal{H}_L)$, respectively. To synchronise again, *later microsteps* are employed which just calculate the active block and use dense-output for the coupling of the latent part.

The propagation of the mixed multirate approach to the DAE network problem (1) necessitated to develop a new class of partitioned B-series to find appropriate coefficients for the scheme (cp. [5]).

Hierarchical mixed multirate allows for an arbitrary amount of activity levels, basically, by replacing the mixed multirate's later micro steps by a series of nested *later compound steps.* To be feasible for industrial applications, each multirate scheme needs a strategy that detects and reacts on events that enforce sudden changes in the local activity. We propose such a procedure considering the inverter chain with 500 inverters as an example.

References

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