MOESP Algorithm for Converting One-dimensional Maxwell Equation into a Linear System

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***Abstract*—In this study, we deal with converting 1-D Maxwell equation to a linear system by using the MOESP(Multivariable Output Error State Space) subspace system identification method. Source is applied to a selected spatial grid point and another spatial grid point is selected as an output i. We collect output data from this spatial point with FDTD algorithm. With this output and input data, required data matrices are built and a SISO (Single Input Single Output) linear system is estimated by MOESP algorithm for 1-D Maxwell equations. The order of the estimated system mainly depends on the structure of the data matrices.**

Keywords—Maxwell Equations, System Identification, Model Order Reduction, MOESP, Mathematical Modelling

I. Introduction

In general, system identification methods mainly developed in the area of automatic control to determine the best (in the sense of input-output relationship) model from a given observed input-output data set. In this study, the Maxwell equation is converted into a set of state-space equations by using MOESP algorithm, which is a member of subspace system identifacation family of algorithms. This idea may be useful when simulation of the VLSI interconnections are considered. To computation of the effects VLSI interconnection is mainly based on the solution of Maxwell equations on chip geometries. The RLC parasitic circuits are realized with the solution of Maxwell equations. Finally, the model order reduction algorithms are implemented to reduce the dimension of the linear subsystem of this RLC circuits [1]. In this study, 1-D Maxwell equation is directly converted into a small order SISO system without using any model reduction algorithm. Therefore it may be useful to find an appropriate reduction order of the model order reduction process. Most important criterion for succesfull convertion is the input-output data set, which produced from 1-D Maxwell equation with the FDTD method.

The remaining of the paper, organized as follows. In second section, the problem is briefly explained. In third section, the methodology and the MOESP algorithm are introduced. The fourth section contains some numerical results and discussions and finally the last section presents conclusions and future works.

II. Definition of the Problem

Consider an one-dimensional space where there are only variations in the x dimension. Assume that the electric field has only a z component. With Faraday and Ampere's law we could write 1-D Maxwell equations as,

$$\mu\frac{\partial H_y}{\partial t} = \frac{\partial E_z}{\partial x},$$
$$\epsilon\frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x}. \tag{1}$$

The source function is applied on to the 0^{th} node of the computational domain and data is collected as the electrical field of 50^{th} node.

After discretization, FDTD algorithm is implemented to obtain the input data u_k and output data y_k.

Two Hankel matrices could be determined in terms of u_k and y_k to generalize the structure. Here the structure of the $U_{0|k-1}$ matrix is showed. The other matrices from the u_k and y_k could be produced similarly.

$$U_{0|k-1} = \begin{bmatrix} u(0) & u(1) & \dots & u(N-1) \\ u(1) & u(2) & \dots & u(N) \\ \vdots & \vdots & & \vdots \\ u(k-1) & u(k) & \dots & u(k+N-2) \end{bmatrix} \in \mathcal{R}^{kpxN} \tag{2}$$

where k is greater than the order of the system(n), p is the number of the outputs of the system, m is the number of the inputs and finally N is a sufficiently large number for fixing the Hankel matrix. 0 and $k-1$ values in Hankel matrices definitions are used for determining the upper-left and lower-left elements respectivly.

LQ decomposition, which is the dual of the QR decomposition, is used to make the upper-right block of the data matrix a zero matrix. LQ decomposition of a data matrix can be given as,

$$\begin{bmatrix} U_{0|k-1} \\ Y_{0|k-1} \end{bmatrix} = \begin{bmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} Q_1^T \\ Q_2^T \end{bmatrix} \tag{3}$$

The actual computation of LQ decomposition is performed by taking transpose of the QR decomposition of the data matrix [2].

III. MOESP Algorithm

LQ decomposition and the SVD are employed in the MOESP algorithm [3]. The algorithm of the MOESP is listed below.

1: Compute the LQ decomposition of (3).
2: Compute the SVD of the L_{22} as,

$$L_{22} = [U_1 U_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix} = U_1 \Sigma_1 V_1^T$$

and determine the order of the system $n = dim(\Sigma_1)$ from the non-zero singular values and define the extended observability matrix of the system as $\mathcal{O}_k = U_1 \Sigma_1^{1/2}$.
3: Obtain C and A from the $\mathcal{O}_k$.
4: Estimate B and D.

After obtaining the A, B, C and D matrices we can represent the 1-D Maxwell equation as a linear system,

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \quad (4)$$

IV. Numerical Examples

In experiments, it is observed that the accuracy of the estimated model and selection of the true order for the system depends on the information that data matrices contain. For example if $u(t) = cos(0.1t)$ the order of the estimated system could be selected as 2 which can be seen from the singular value distribution that given in Fig. 2. But, if u(t) selected as a constant source, for example $u(t) = 10$, because of the structure of data matrices it is not possible to find an accurate estimation. To solve this problem data set can be reduced with a random selection from original data. After reducing the number of data set more accurate results are obtained from the algorithm. This situation is showed in Figs. 3 and 4.

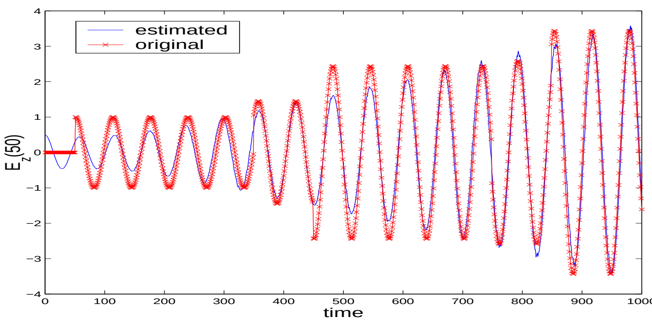

Fig. 1. Original and estimated outputs($E_z(50)$) for u(t)=cos(0.1t) where estimated system order equals to 2

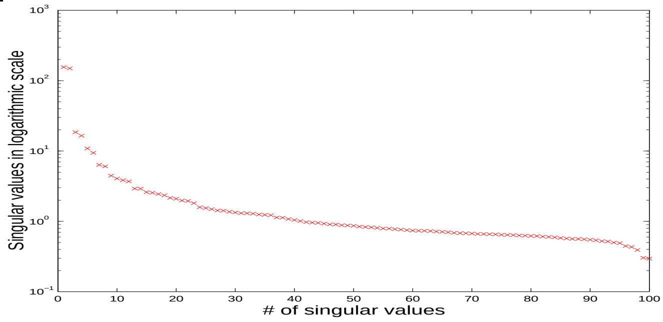

Fig. 2. Singular value distribution of L_{22} matrix where u(t)=cos(0.1t)

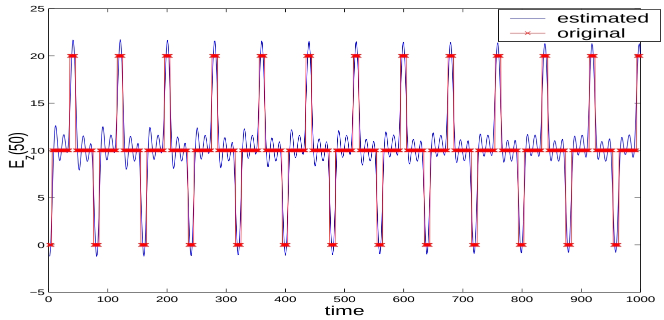

Fig. 3. Original and Estimated outputs($E_z(50)$) with sampled case where u(t)=10 and estimated system order equals to 6

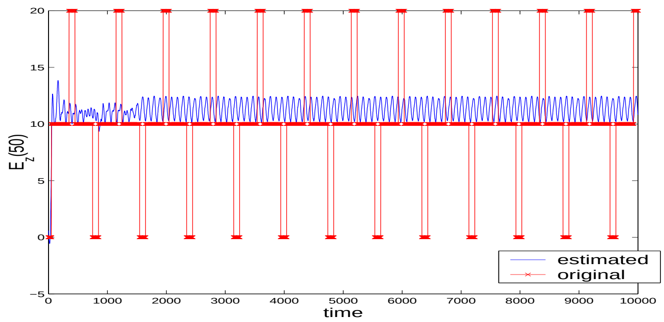

Fig. 4. Original and Estimated outputs($E_z(50)$) with unsampled case where u(t)=10 and estimated system order equals to 6

V. Conclusion and Future Works

In this paper, 1-D Maxwell equation converts into a SISO linear state-space system with the MOESP algorithm. Some important observations are made from the numerical experiments. The mathematical properties of the source function is very important for the accuracy of the method. For constant source case the rank of the data matrix is mainly determined by the output data. Therefore sampling of the data is required for more accurate results. But in all cases, estimated output have some numerical distortion. To achieve more accurate result some optimization techniques (neural networks, genetic algorithms, etc.) can be implemented.

The future works will be focused on the building a relationship between the determining the order for estimated system and the properties of data matrices and extending the method to MIMO (Multiple Input Multiple Output) cases.

References

[1] Athanasios C. Antoluas, *Approximation of Large-Scale Dynamical Systems*, SIAM, 2005.
[2] Edward Ciggaar, "The moesp algorithm for linear time domain modelling," *Nat. Lab. Technical Note TN 123/98*, 1998.
[3] Tohru Katayama, *Subspace Methods for System Identification*, Springer-Verlag, 2005.